

# Quarkonium Production within Jets

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# Fragmenting Jet Functions (FJFs)

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## NRQCD and Quarkonium Production

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## Heavy Quarkonium FJFs

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## Recent Data on Quarkonia in Jets (LHCb)

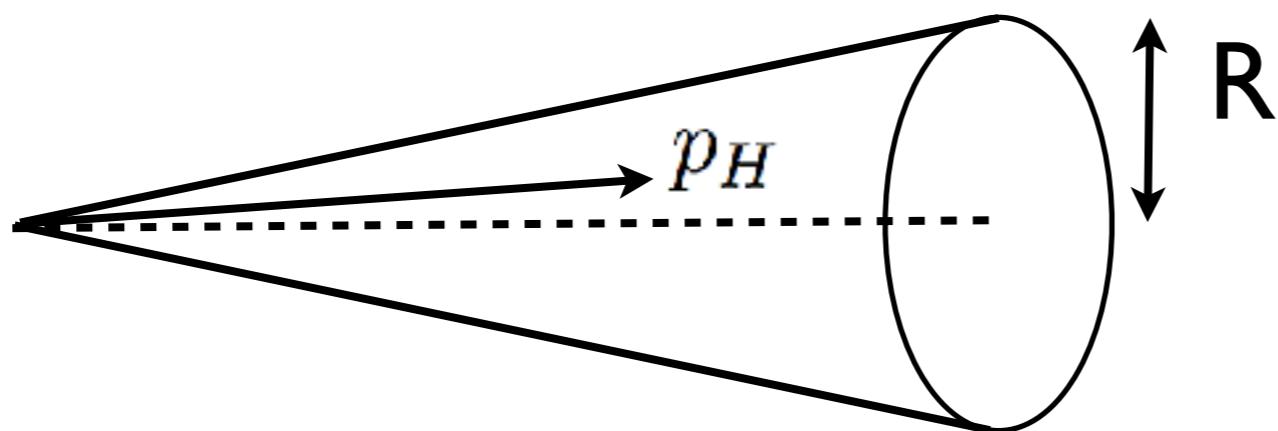
# Fragmenting Jet Functions

M. Procura, I. Stewart, PRD 81 (2010) 074009

A. Jain, M. Procura, W. Waalewijn, JHEP 1105 (2011) 035

A. Procura, W. Waalewijn, PRD 85 (2012) 114041

jets with identified hadrons



Jet Energy:  $E$   
 $p_H^+ = z p_{\text{jet}}^+$

cross sections determined by **fragmenting jet function (FJF)**:

$$\mathcal{G}_g^h(E, R, \mu, z)$$

inclusive hadron production: fragmentation functions

$$\frac{1}{\sigma_0} \frac{d\sigma^h}{dz}(e^+e^- \rightarrow h X) = \sum_i \int_z^1 \frac{dx}{x} C_i(E_{\text{cm}}, x, \mu) D_i^h(z/x, \mu)$$

jet cross sections: jet functions

$$d\sigma(E, R) = \int d\Phi_N \text{tr}[H_N S_N] \prod_\ell J_\ell$$

$$\mathcal{G}_g^h(E, R, \mu, z) \longrightarrow D_i^h(z/x, \mu), J_\ell$$

relationship to jet function:

$$\sum_h \int_0^1 dz z D_j^h(z, \mu) = 1$$

→  $J_i(E, R, z, \mu) = \frac{1}{2} \sum_h \int \frac{dz}{(2\pi)^3} z \mathcal{G}_i^h(E, R, z, \mu)$

cross section for jet w/ identified hadron from jet cross section

$$\frac{d\sigma}{dE} = \int d\Phi_N \text{tr}[H_N S_N] \prod_\ell J_\ell J_i(E, R, \mu)$$

→  $\frac{d\sigma}{dEdz} = \int d\Phi_N \text{tr}[H_N S_N] \prod_\ell J_\ell \mathcal{G}_i^h(E, R, z, \mu)$

relationship to fragmentation functions

$$\mathcal{G}_i^h(E, R, z, \mu) = \sum_i \int_z^1 \frac{dz'}{z'} \mathcal{J}_{ij}(E, R, z', \mu) D_j^h\left(\frac{z}{z'}, \mu\right) \left[ 1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{4E^2 \tan^2(R/2)}\right) \right]$$

**matching coefficients calculable in perturbation theory**

$$\frac{\mathcal{J}_{gg}(E, R, z, \mu)}{2(2\pi)^3} = \delta(1-z) + \frac{\alpha_s(\mu) C_A}{\pi} \left[ \left( L^2 - \frac{\pi^2}{24} \right) \delta(1-z) + \hat{P}_{gg}(z)L + \hat{\mathcal{J}}_{gg}(z) \right]$$

$$\hat{\mathcal{J}}_{gg}(z) = \begin{cases} \frac{\hat{P}_{gg}(z) \ln z}{z} & z \leq 1/2 \\ \frac{2(1-z+z^2)^2}{z} \left( \frac{\ln(1-z)}{1-z} \right)_+ & z \geq 1/2. \end{cases}$$

$$L = \ln[2E \tan(R/2)/\mu].$$

scale for  $\mathcal{J}_{ij}(E, R, z, \mu)$

sum rule for matching coefficients

$$\sum_j \int_0^1 dz z \mathcal{J}_{ij}(R, z, \mu) = 2(2\pi)^3 J_i(R, \mu)$$

# Non-Relativistic QCD (NRQCD) Factorization Formalism

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Bodwin, Braaten, Lepage, PRD 51 (1995) 1125

$$\sigma(gg \rightarrow J/\psi + X) = \sum_n \sigma(gg \rightarrow c\bar{c}(n) + X) \langle \mathcal{O}^{J/\psi}(n) \rangle$$
$$n = {}^{2S+1}L_J^{(1,8)}$$

double expansion in  $\alpha_s, v$

## NRQCD long-distance matrix element (LDME)

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$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle \sim v^3$$

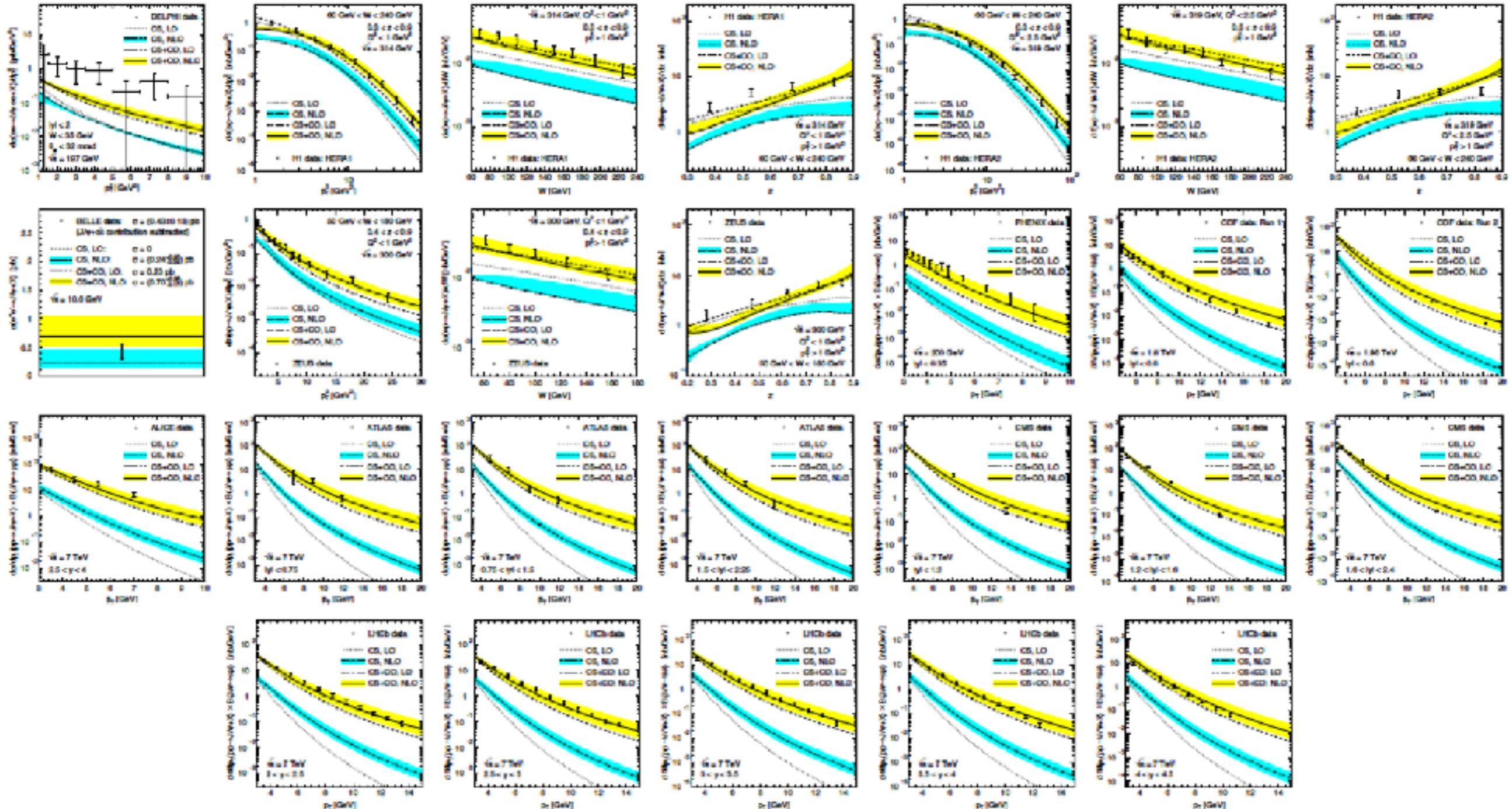
CSM - lowest order in  $v$

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle, \langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle, \langle \mathcal{O}^{J/\psi}(^3P_J^{[8]}) \rangle \sim v^7$$

color-octet mechanisms

# Global Fits with NLO CSM + COM

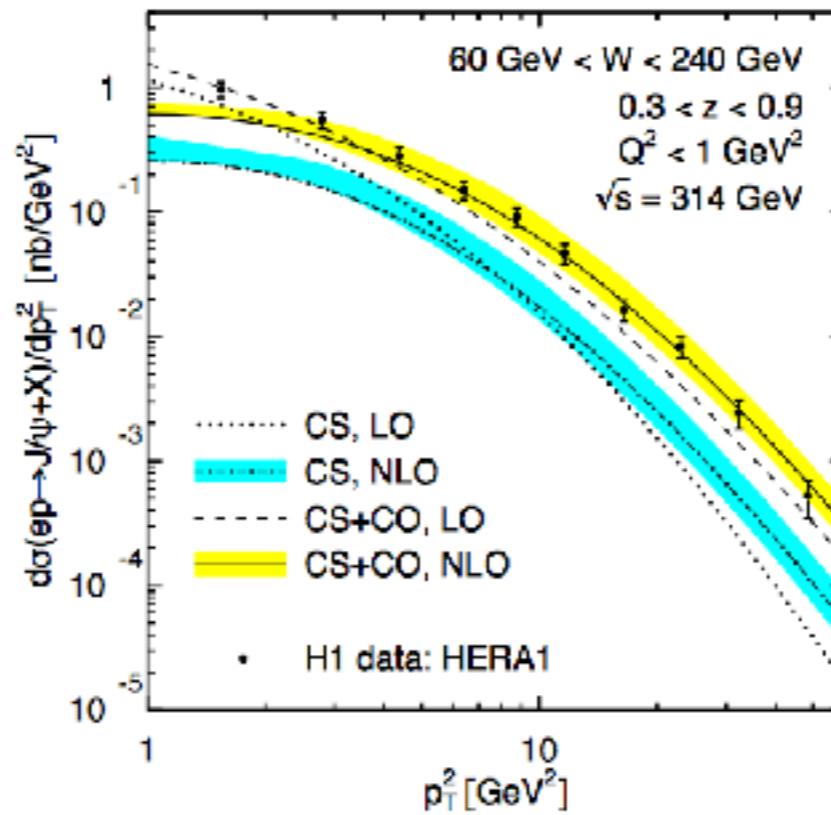
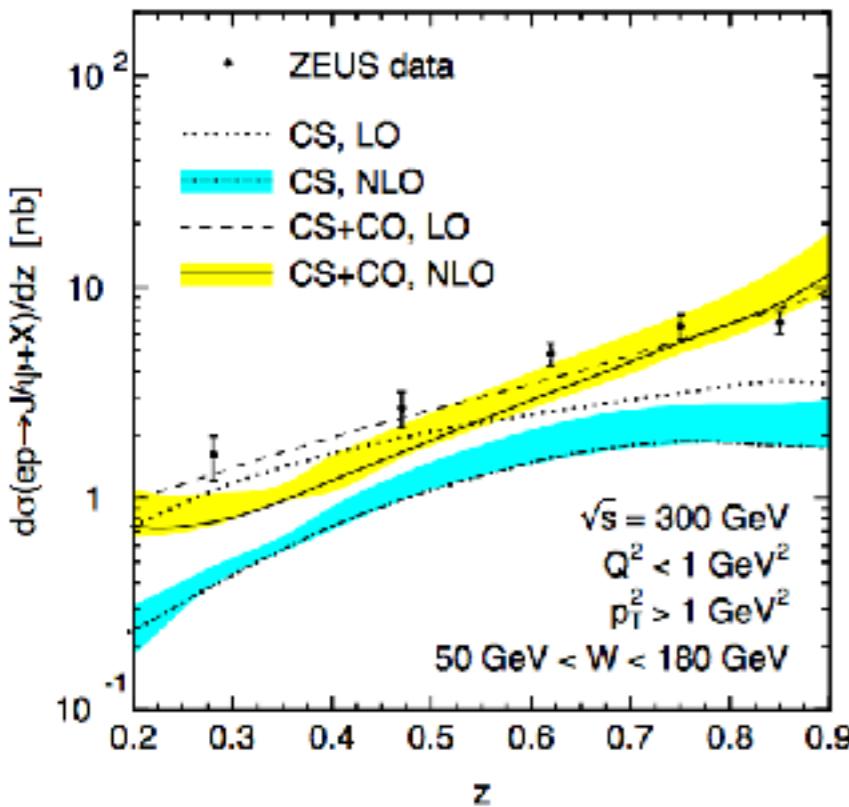
Butenschoen and Kniehl, PRD 84 (2011) 051501



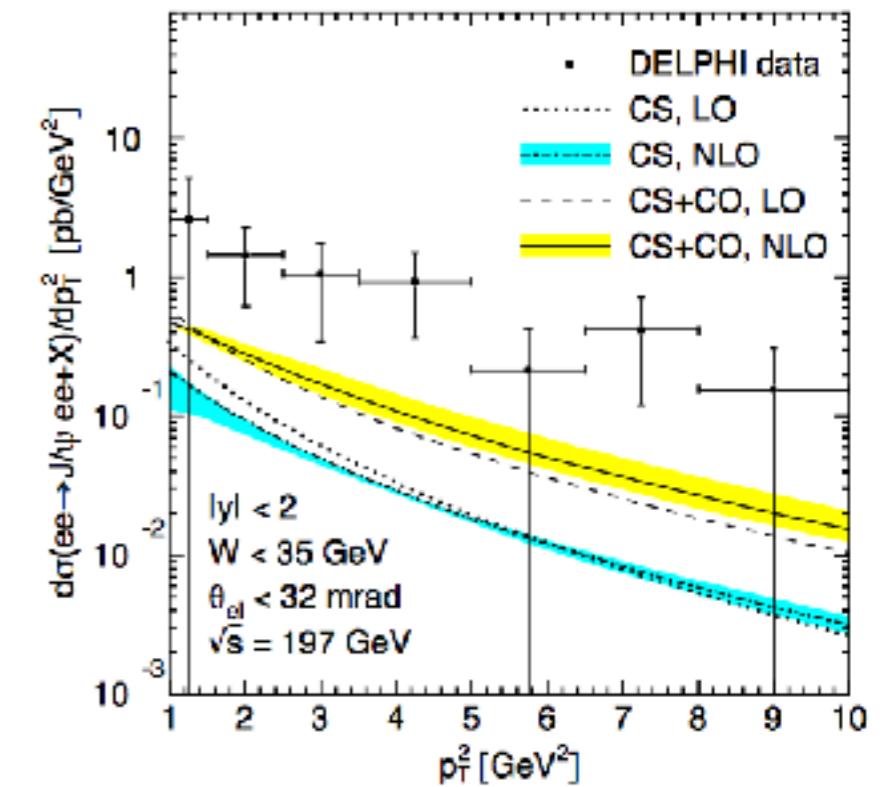
$e^+e^-$ ,  $\gamma\gamma$ ,  $\gamma p$ ,  $p\bar{p}$ ,  $pp \rightarrow J/\psi + X$

fit to 194 data points, 26 data sets

# NLO: CSM + COM Required to Fit Data



$ep \rightarrow J/\psi + X$



$\gamma^* \gamma^* \rightarrow J/\psi + X$

# Status of NRQCD approach to J/ψ Production

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NLO: COM + CSM required for most processes

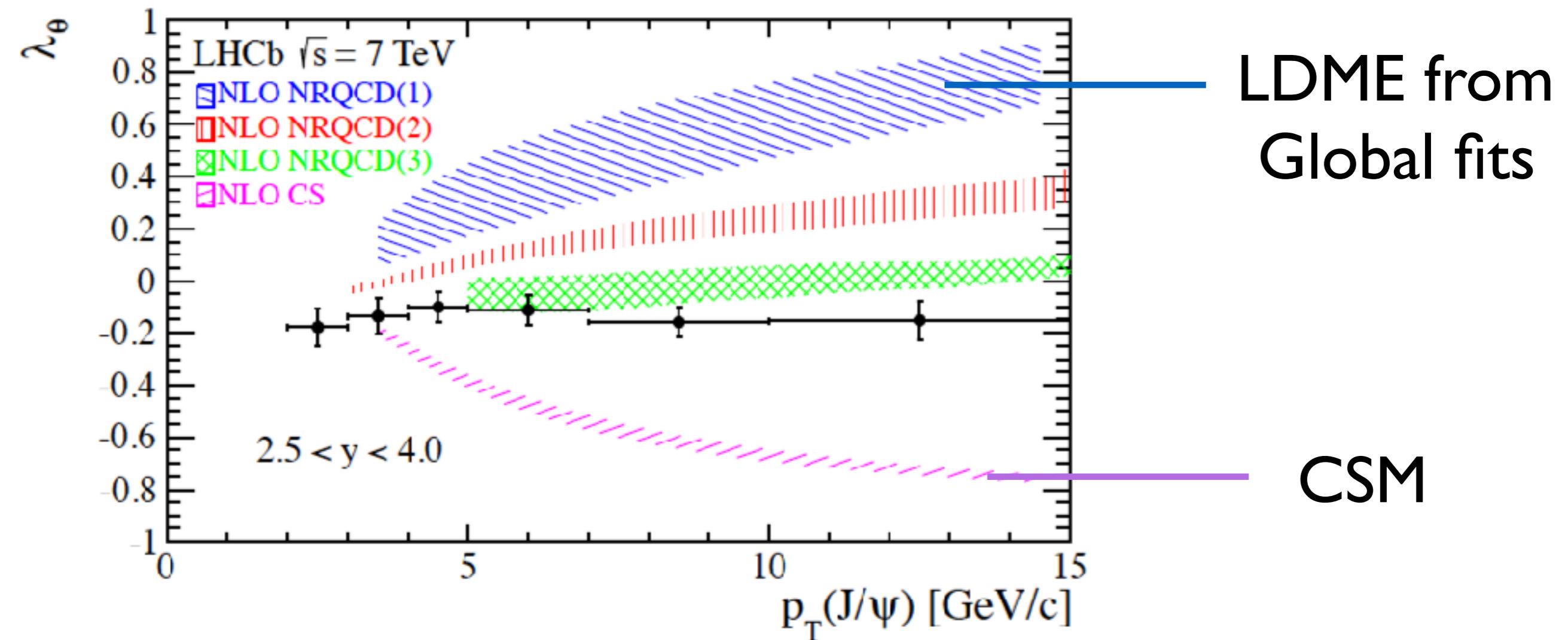
**extracted LDME satisfy NRQCD v-scaling**

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle = 1.32 \text{ GeV}^3$$

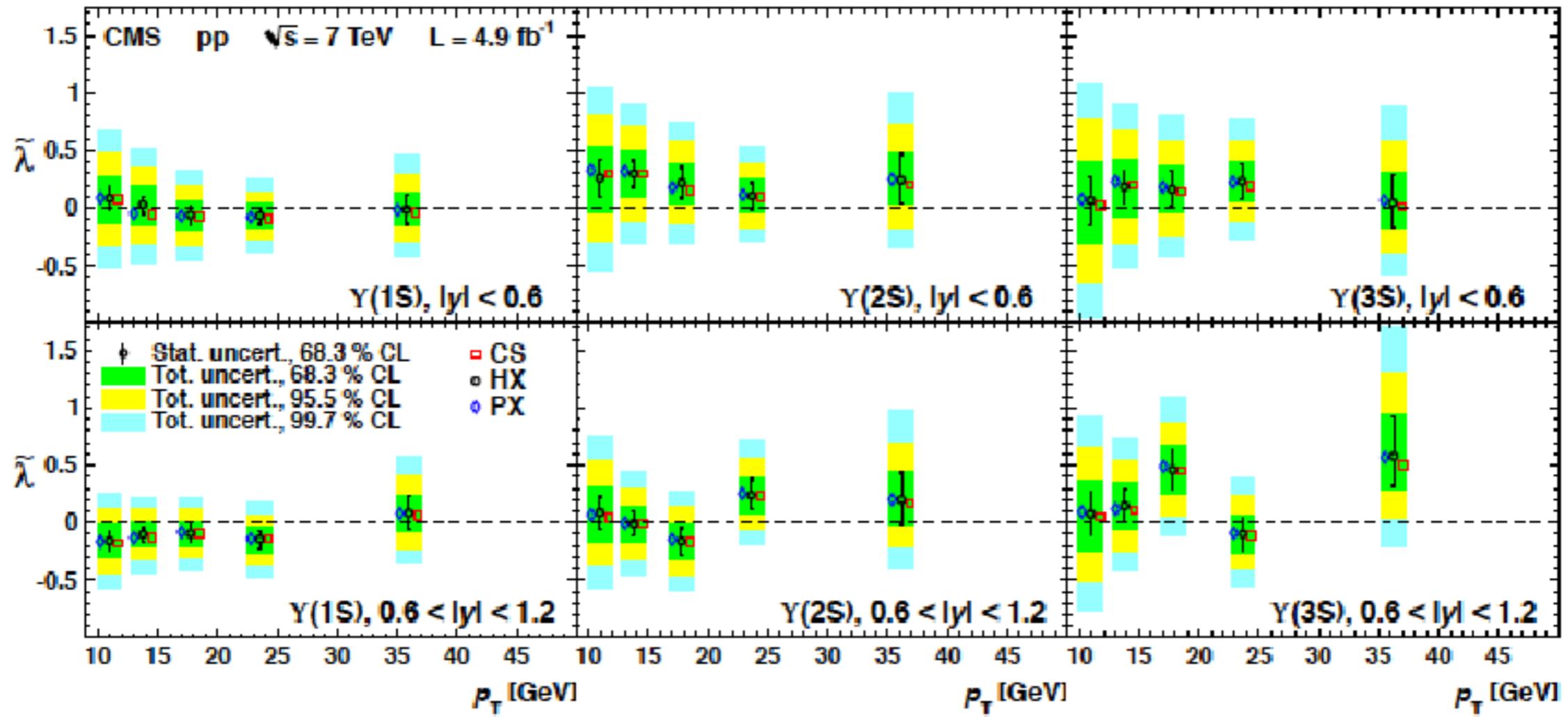
$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$	$(4.97 \pm 0.44) \times 10^{-2} \text{ GeV}^3$
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$	$(2.24 \pm 0.59) \times 10^{-3} \text{ GeV}^3$
$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$	$(-1.61 \pm 0.20) \times 10^{-2} \text{ GeV}^5$

$$\chi^2_{\text{d.o.f.}} = 857/194 = 4.42$$

# Polarization of J/ $\psi$ at LHCb



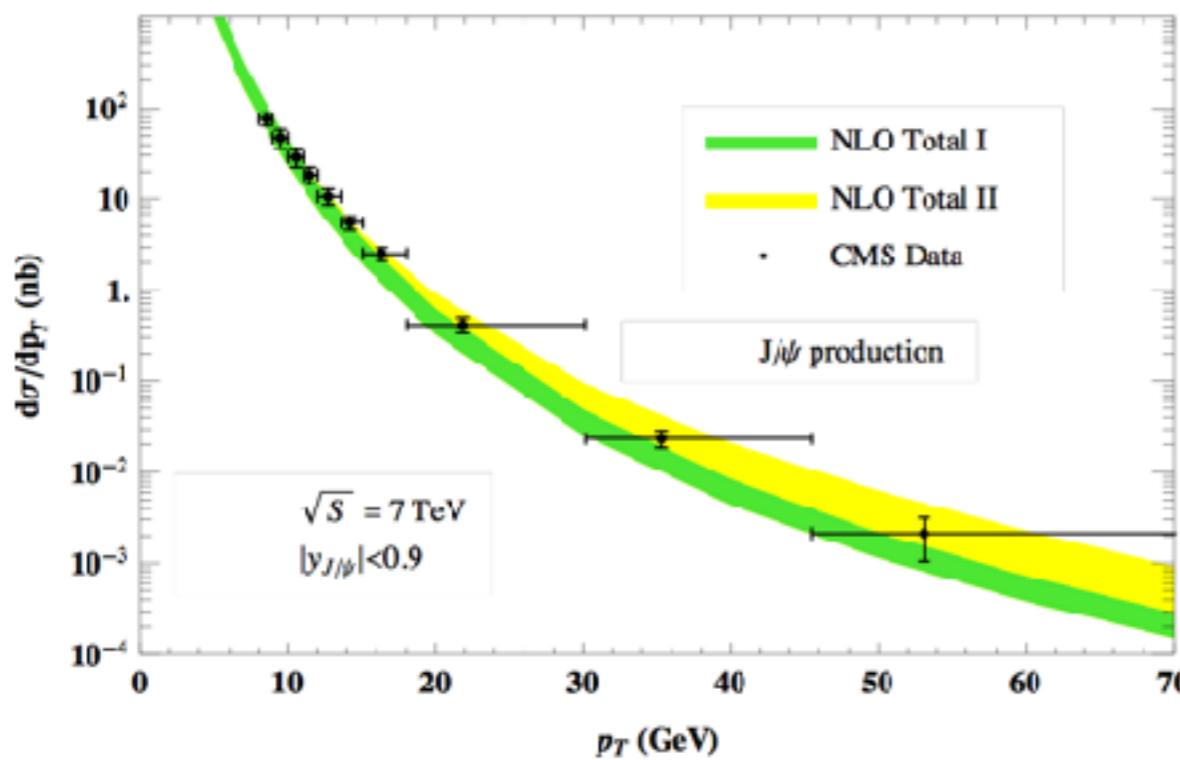
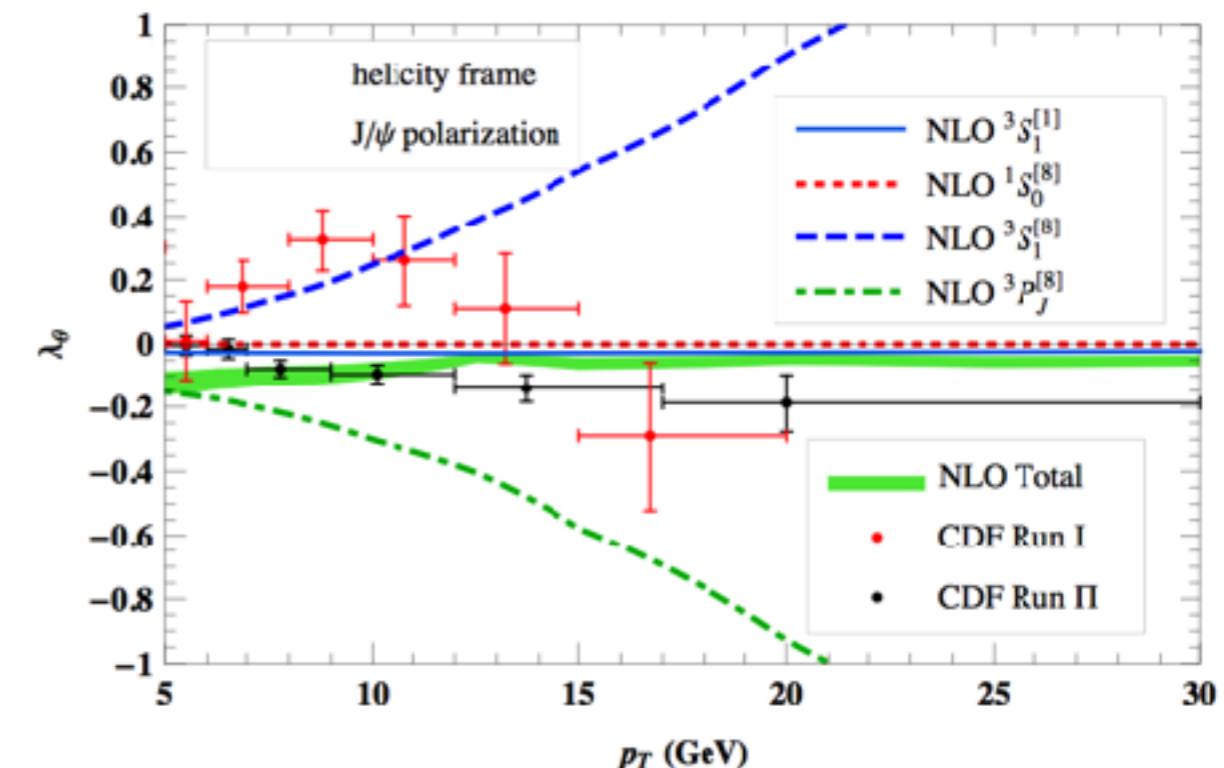
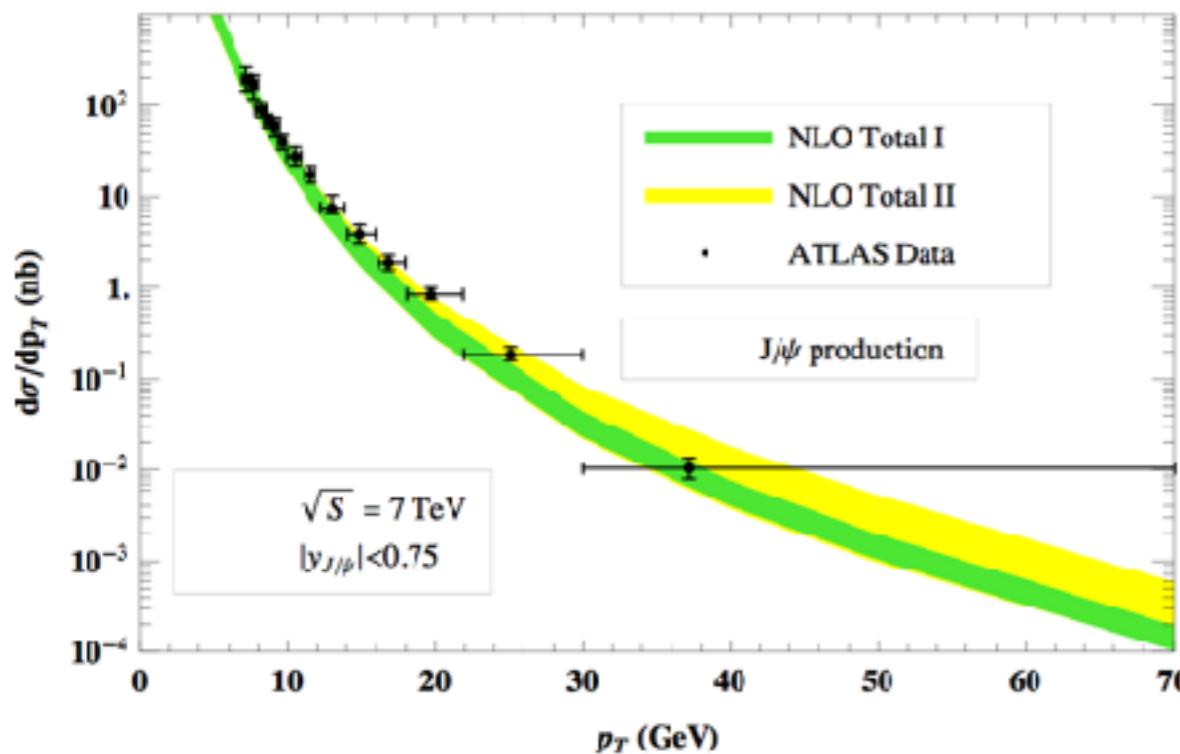
# Polarization of $\Upsilon(nS)$ at CMS



# Recent Attempts to Resolve J/ $\psi$ Polarization Puzzle

simultaneous NLO fit to CMS,ATLAS high  $p_T$  production, polarization

Chao, et. al. PRL 108, 242004 (2012)



$\langle \mathcal{O}(^3S_1^{[1]}) \rangle$	$\langle \mathcal{O}(^1S_0^{[8]}) \rangle$	$\langle \mathcal{O}(^3S_1^{[8]}) \rangle$	$\langle \mathcal{O}(^3P_0^{[8]}) \rangle/m_c^2$
GeV <sup>3</sup>	10 <sup>-2</sup> GeV <sup>3</sup>	10 <sup>-2</sup> GeV <sup>3</sup>	10 <sup>-2</sup> GeV <sup>3</sup>
1.16	$8.9 \pm 0.98$	$0.30 \pm 0.12$	$0.56 \pm 0.21$
1.16	0	1.4	2.4
1.16	11	0	0

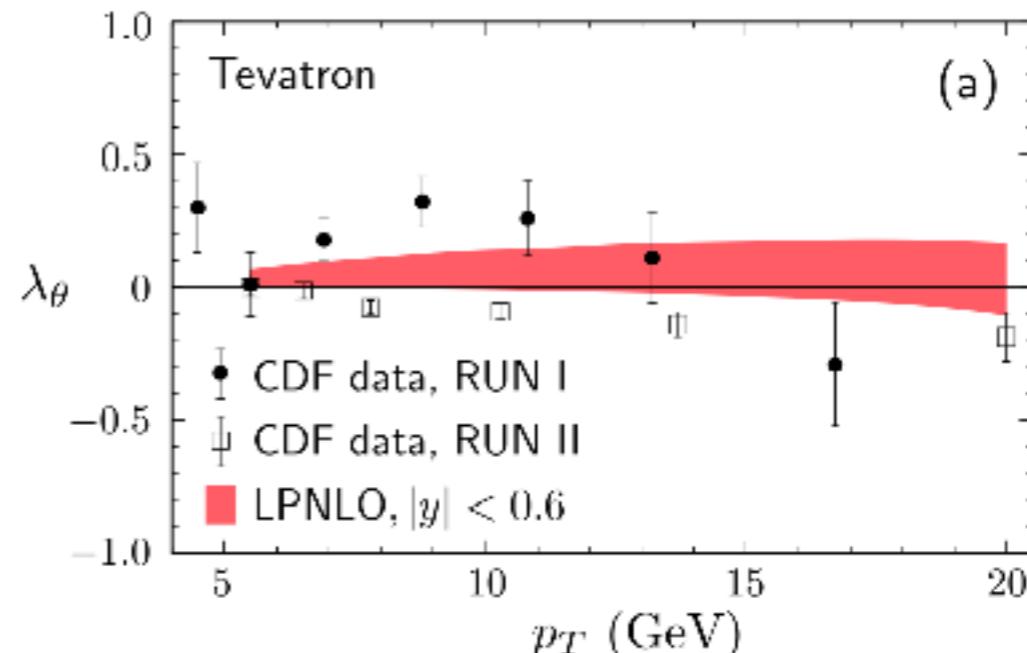
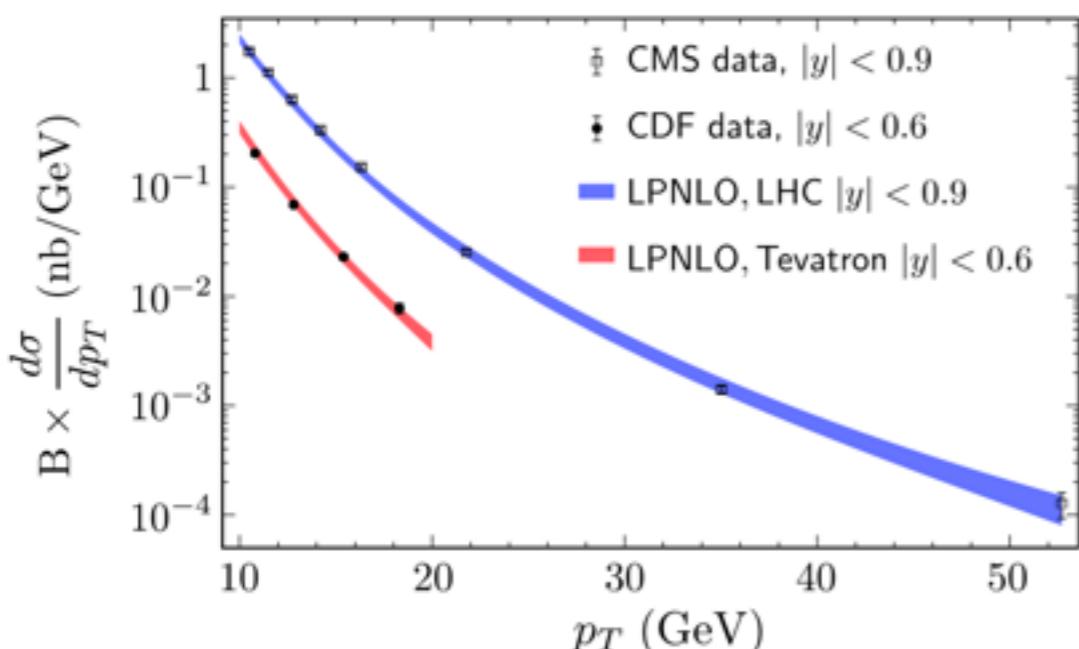
# Recent Attempts to Resolve J/ $\psi$ Polarization Puzzle

i) large  $p_T$  production at CDF

Bodwin, et. al., PRL 113, 022001(2014)

ii) resum logs of  $p_T/m_c$  using AP evolution

iii) fit COME to  $p_T$  spectrum, predict basically no polarization



Extracted COME **inconsistent** with global  
fits

$$\langle \mathcal{O}^{J/\psi}(^1S_0^{(8)}) \rangle = 0.099 \pm 0.022 \text{ GeV}^3$$

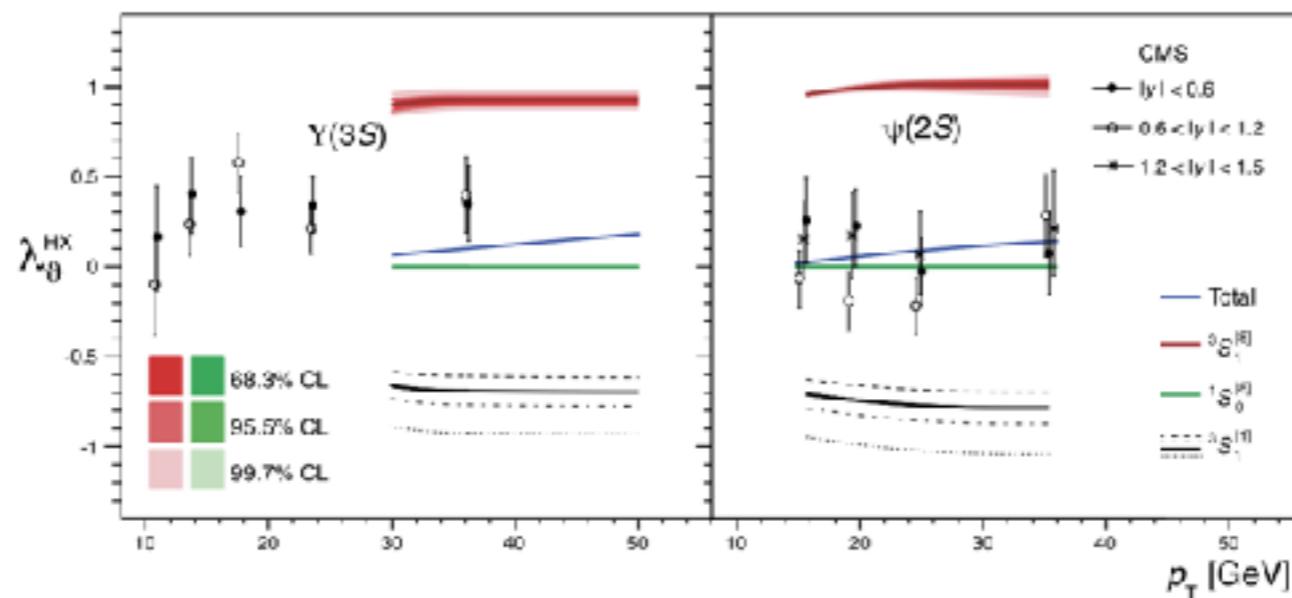
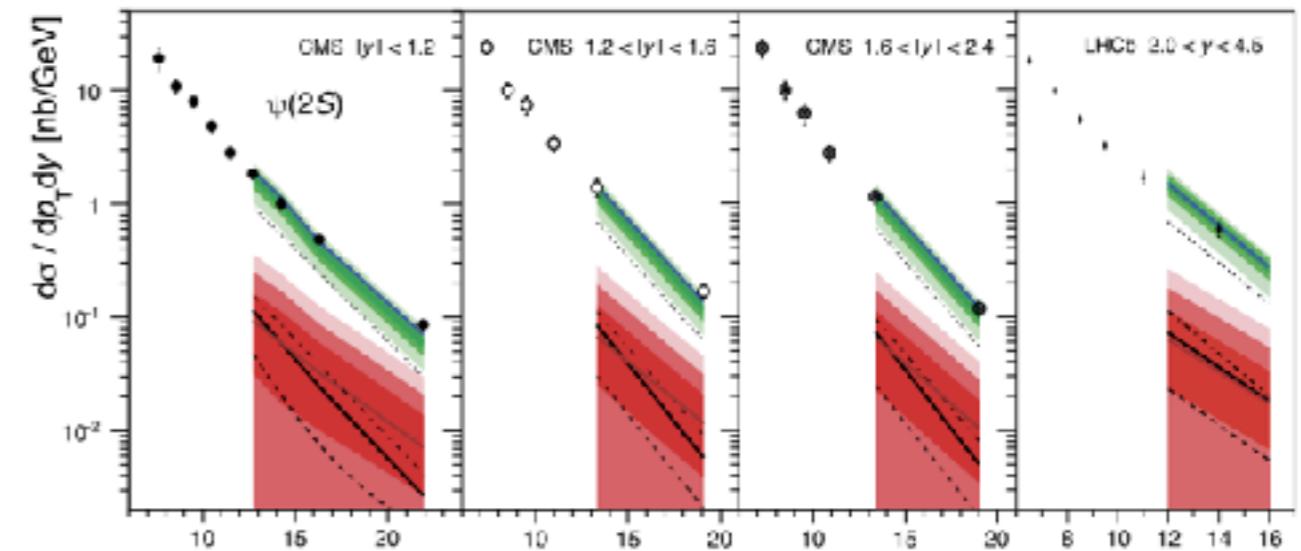
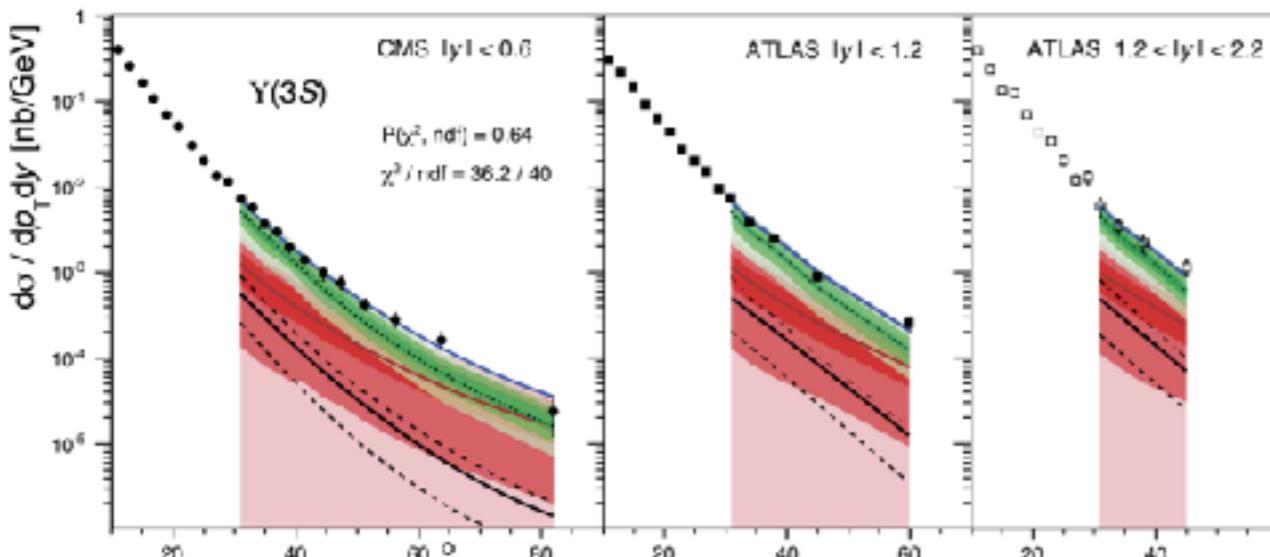
$$\langle \mathcal{O}^{J/\psi}(^3S_1^{(8)}) \rangle = 0.011 \pm 0.010 \text{ GeV}^3$$

$$\langle \mathcal{O}^{J/\psi}(^3P_0^{(8)}) \rangle = 0.011 \pm 0.010 \text{ GeV}^5$$

# Recent Attempts to Resolve J/ $\psi$ Polarization Puzzle

Faccioli, et. al. PLB736 (2014) 98

Lourenco, et. al., NPA, in press



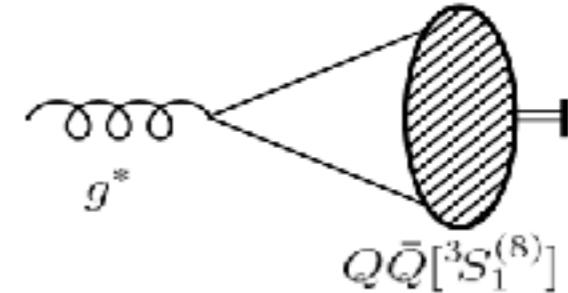
argue for  ${}^1S_0^{[8]}$  dominance in both  $\psi(2S)$  &  $\Upsilon(3S)$  production

# NRQCD fragmentation functions

Braaten, Yuan, PRD 48 (1993) 4230  
Braaten, Chen, PRD 54 (1996) 3216  
Braaten, Fleming, PRL 74 (1995) 3327

Perturbatively calculable **at the scale  $2m_c$**

$$D_g^{\psi(8)}(z, 2m_c) = \frac{\pi \alpha_s(2m_c)}{3M_\psi^3} \langle O^\psi(^3S_1^{(8)}) \rangle \delta(1-z)$$



$$\begin{aligned} D_g^{\psi(1)}(z, 2m_c) &= \frac{5\alpha_s^3(2m_c)}{648\pi^2} \frac{\langle O^\psi(^3S_1^{(1)}) \rangle}{M_\psi^3} \int_0^z dr \int_{(r+z^2)/2z}^{(1+r)/2} dy \frac{1}{(1-y)^2(y-r)^2(y^2-r)^2} \\ &\quad \sum_{i=0}^2 z^i \left( f_i(r, y) + g_i(r, y) \frac{1+r-2y}{2(y-r)\sqrt{y^2-r}} \ln \frac{y-r+\sqrt{y^2-r}}{y-r-\sqrt{y^2-r}} \right), \end{aligned}$$

**Altarelli-Parisi evolution:  $2m_c$  to  $2E$   
 $\tan(R/2)$**

## FJF in terms of fragmentation function

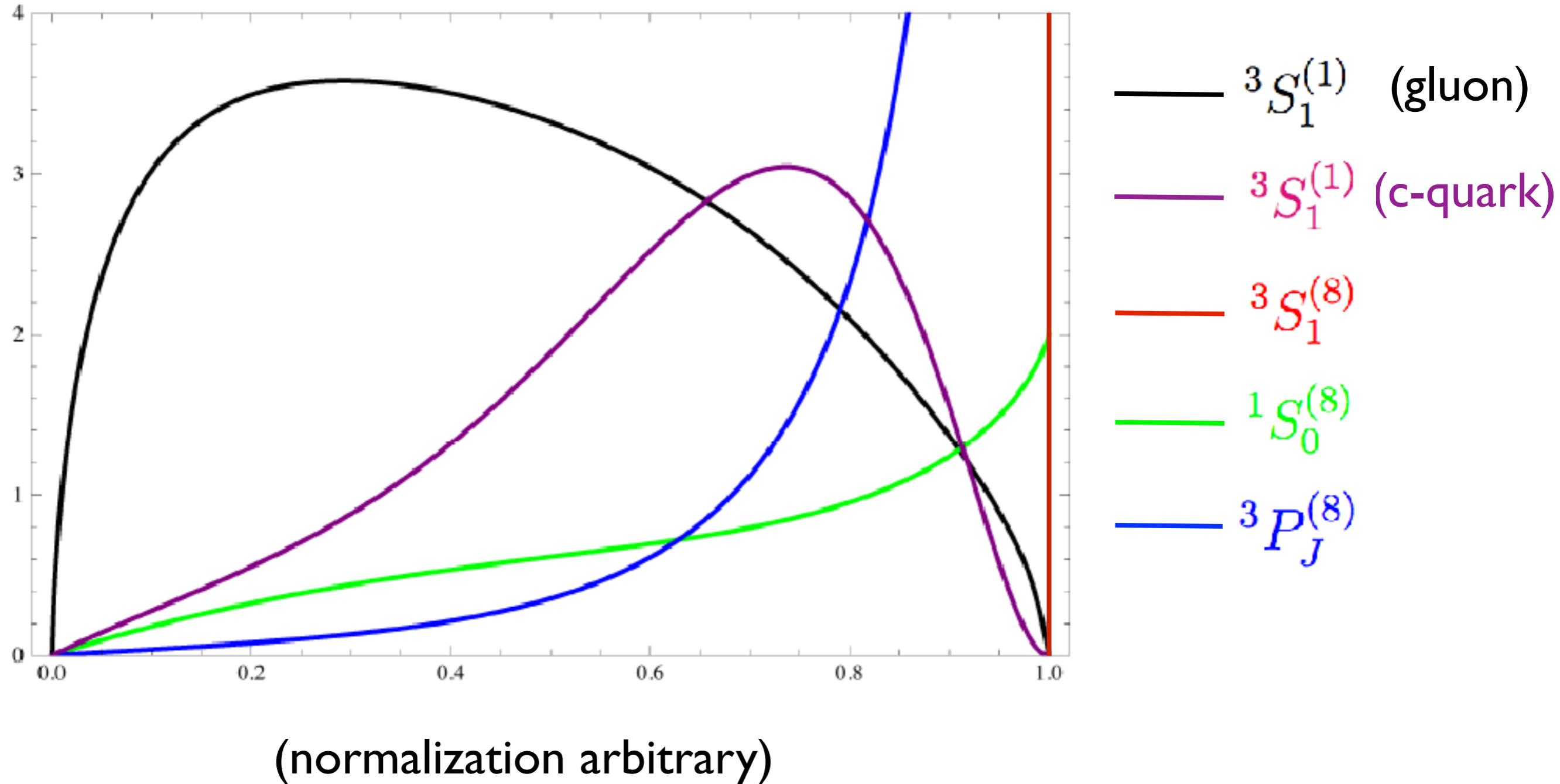
$$\begin{aligned}\mathcal{G}_g^\psi(E, R, z, \mu) = & D_{g \rightarrow \psi}(z, \mu) \left( 1 + \frac{C_A \alpha_s}{\pi} \left( L_{1-z}^2 - \frac{\pi^2}{24} \right) \right) \\ & + \frac{C_A \alpha_s}{\pi} \left[ \int_z^1 \frac{dy}{y} \tilde{P}_{gg}(y) L_{1-y} D_{g \rightarrow \psi} \left( \frac{z}{y}, \mu \right) \right. \\ & + 2 \int_z^1 dy \frac{D_{g \rightarrow \psi}(z/y, \mu) - D_{g \rightarrow \psi}(z, \mu)}{1-y} L_{1-y} \\ & \left. + \theta \left( \frac{1}{2} - z \right) \int_z^{1/2} \frac{dy}{y} \hat{P}_{gg}(y) \ln \left( \frac{y}{1-y} \right) D_{g \rightarrow \psi} \left( \frac{z}{y}, \mu \right) \right]\end{aligned}$$

$$L_{1-z} = \ln \left( \frac{2E \tan(R/2)(1-z)}{\mu} \right)$$

**For large E, FJF  $\sim$  NRQCD frag. function (at scale  $2E \tan(R/2)$ )**

$$\mathcal{G}_g^h(E, R, \mu = 2E \tan(R/2), z) \rightarrow D_g^\psi(z, 2E \tan(R/2)) + O(\alpha_s)$$

# NRQCD FF's (at scale $2m_c$ )

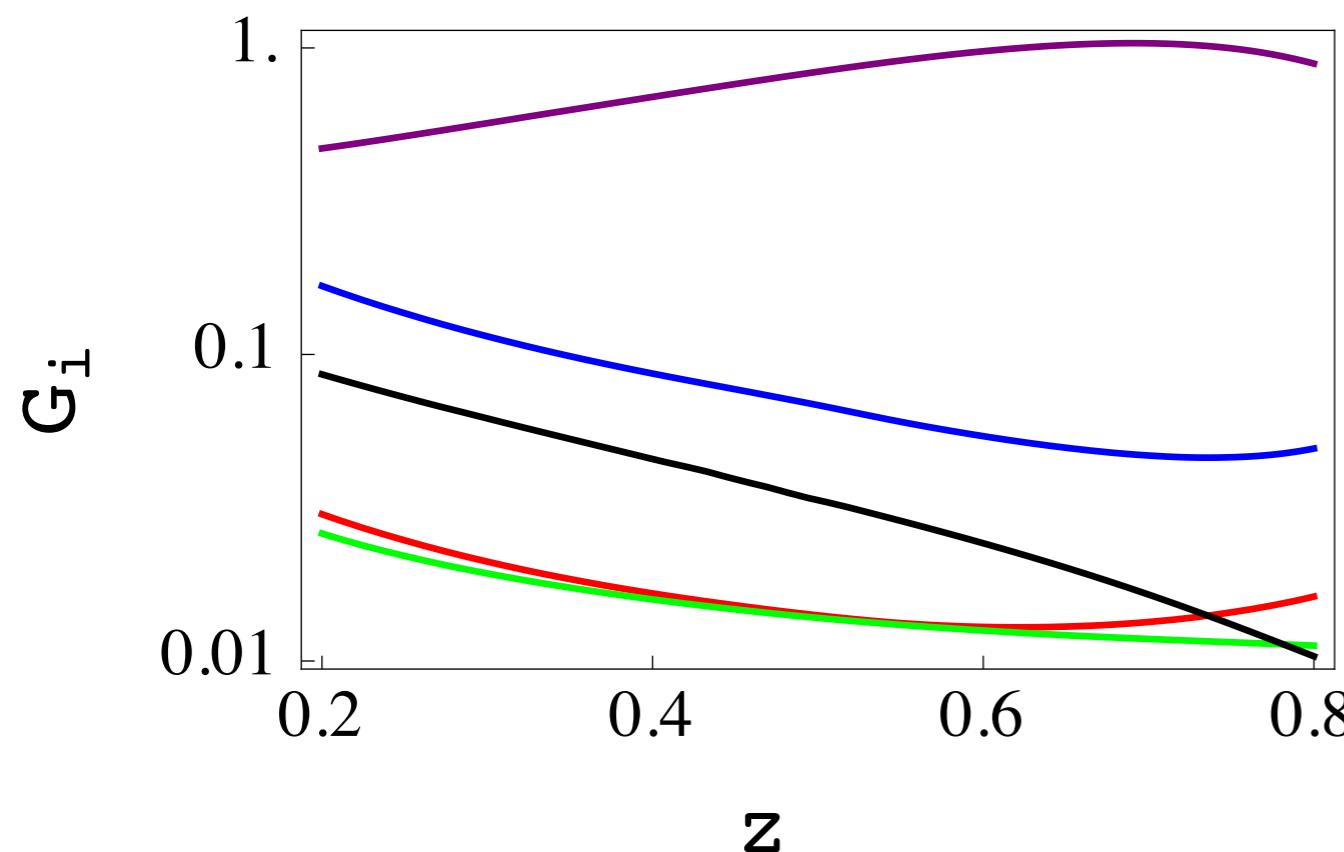


Evolution to  $2E \tan(R/2)$  will soften discrepancies

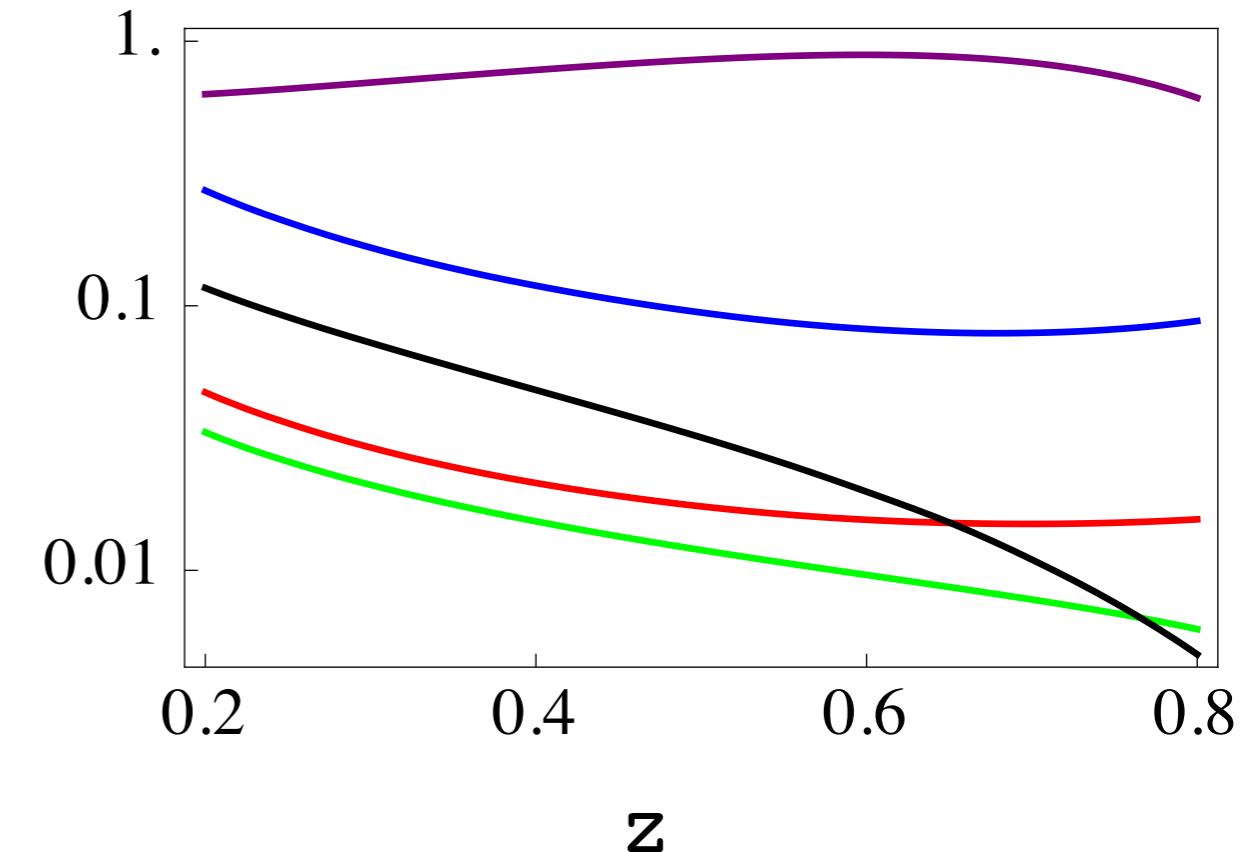
# FJF's at Fixed Energy vs. z

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003

$E = 50 \text{ GeV}$

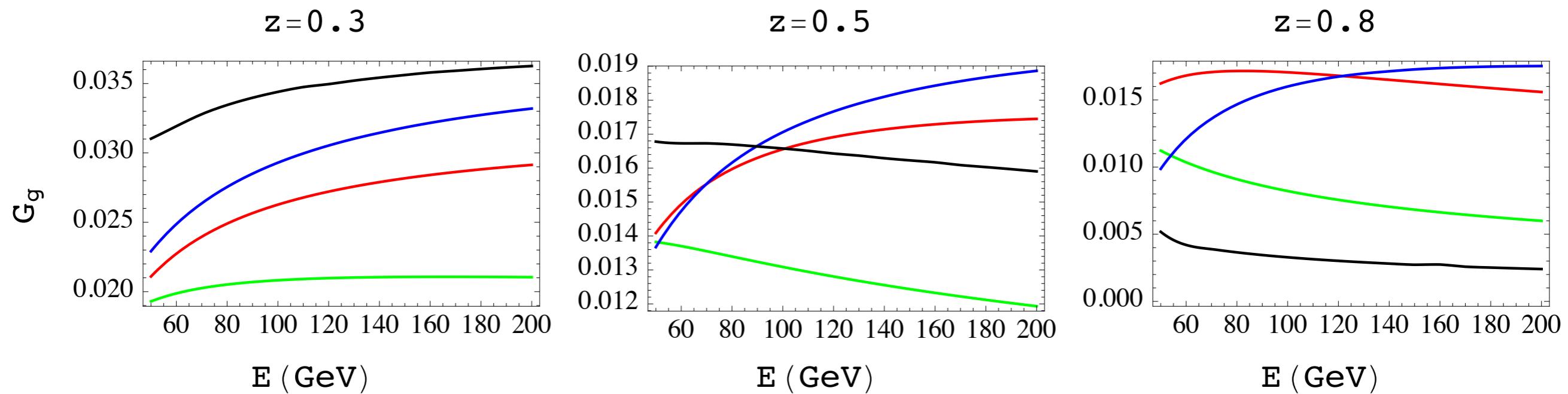


$E = 200 \text{ GeV}$



# FJF's at Fixed z vs. Energy

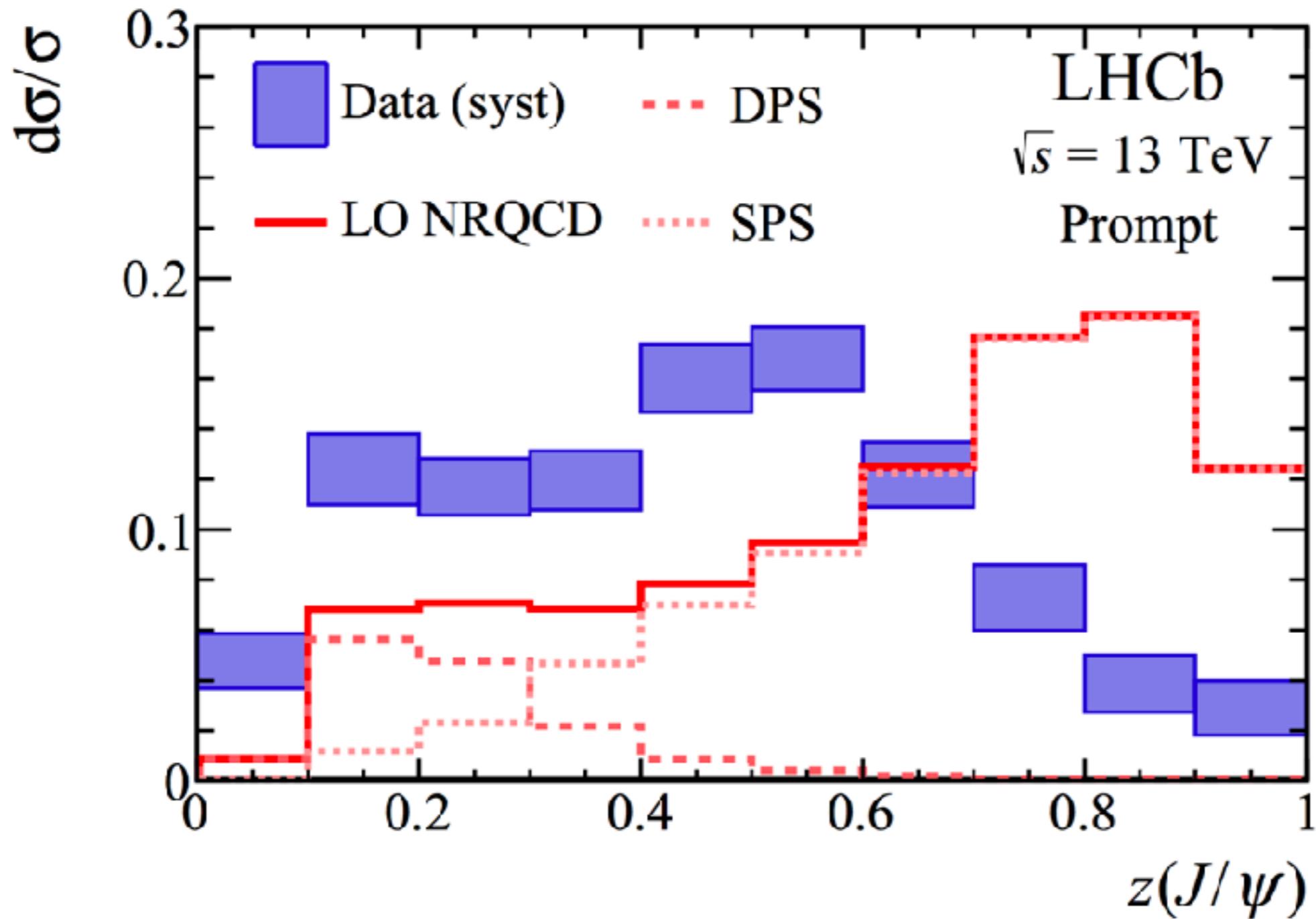
M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003



${}^1S_0^{(8)}$  dominance predicts negative slope for  $z$  vs.  $E$  if  $z > 0.5$

# Recent Observations of Quarkonia within Jets

LHCb collaboration, Phys. Rev. Lett. 118 (2017) no.19, 192001



**cuts:**  $2.5 < \eta_{\text{jet}} < 4.0$     $p_{T,jet} > 20 \text{ GeV}$     $p(\mu) > 5 \text{ GeV}$

This result was anticipated in:

## Jets w/ Heavy Mesons: NLL' vs. Monte Carlo

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(w/ R. Bain, L. Dai, A. Hornig, A. Leibovich, Y. Makris)

JHEP 1606 (2016) 121 (arXiv:1601.05815)

$$e^+ e^- \rightarrow b\bar{b}$$

↳ B jet

$$e^+ e^- \rightarrow q\bar{q}g$$

↳ J/ψ jet

# $e^+e^- \rightarrow$ Jets in SCET

S.D. Ellis, et.al., JHEP1011(2010) 101

$$d\sigma = H \times J_q \otimes J_{\bar{q}} \otimes J_g \otimes S$$

$$\longrightarrow d\sigma = H \times J_q \otimes J_{\bar{q}} \otimes \mathcal{G}_g^{J/\psi} \otimes S$$

unmeasured jets:

E, R

measured jets:

angularity:  $\tau_a = \frac{1}{\omega} \sum_i (p_i^+)^{1-a/2} (p_i^-)^{a/2}$

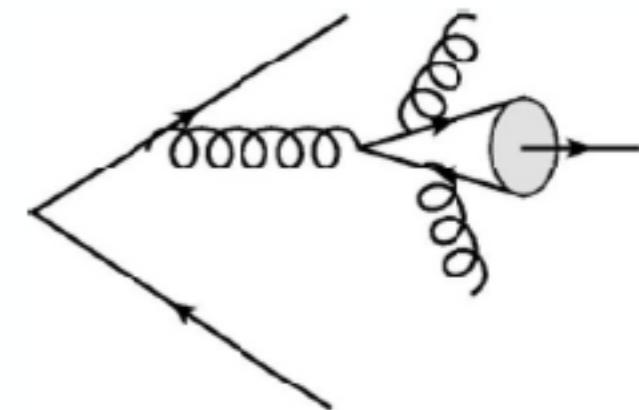
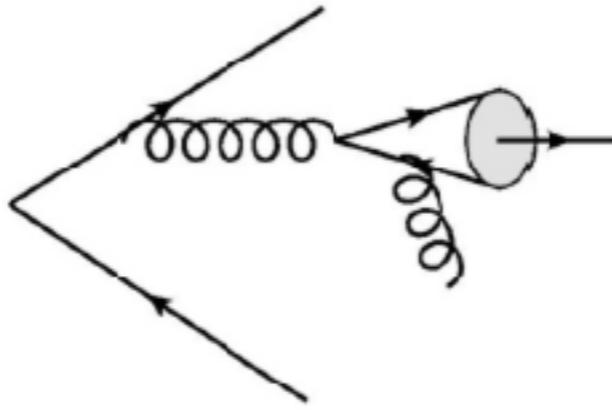
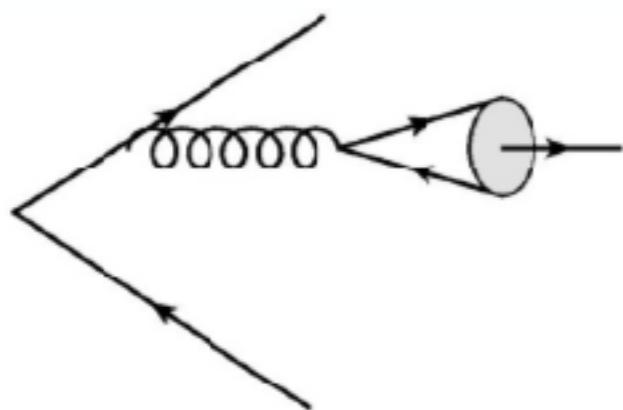
$$\omega = \sum_i p_i^- \quad s = \omega^2 \tau_0$$

# Madgraph + PYTHIA

$$e^+e^- \rightarrow b\bar{b}c\bar{c} [{}^3S_1^{(8)}]$$

$$e^+e^- \rightarrow b\bar{b}g c\bar{c} [{}^1S_0^{(8)}]$$

$$e^+e^- \rightarrow b\bar{b}ggc\bar{c} [{}^3S_1^{(1)}]$$

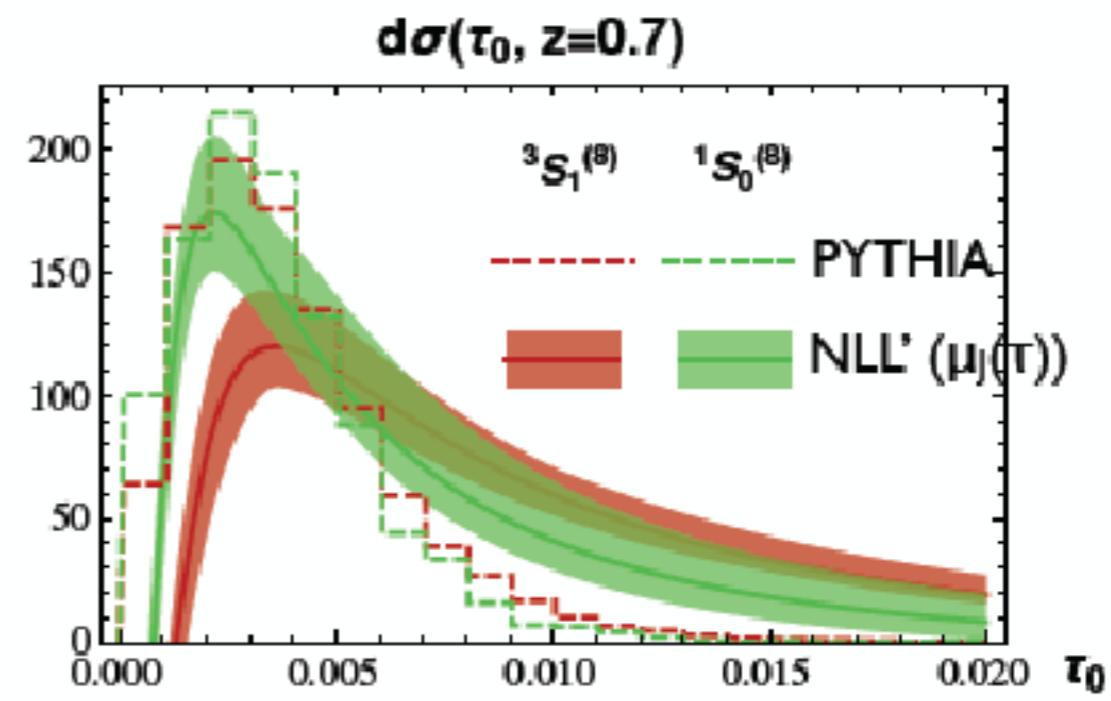
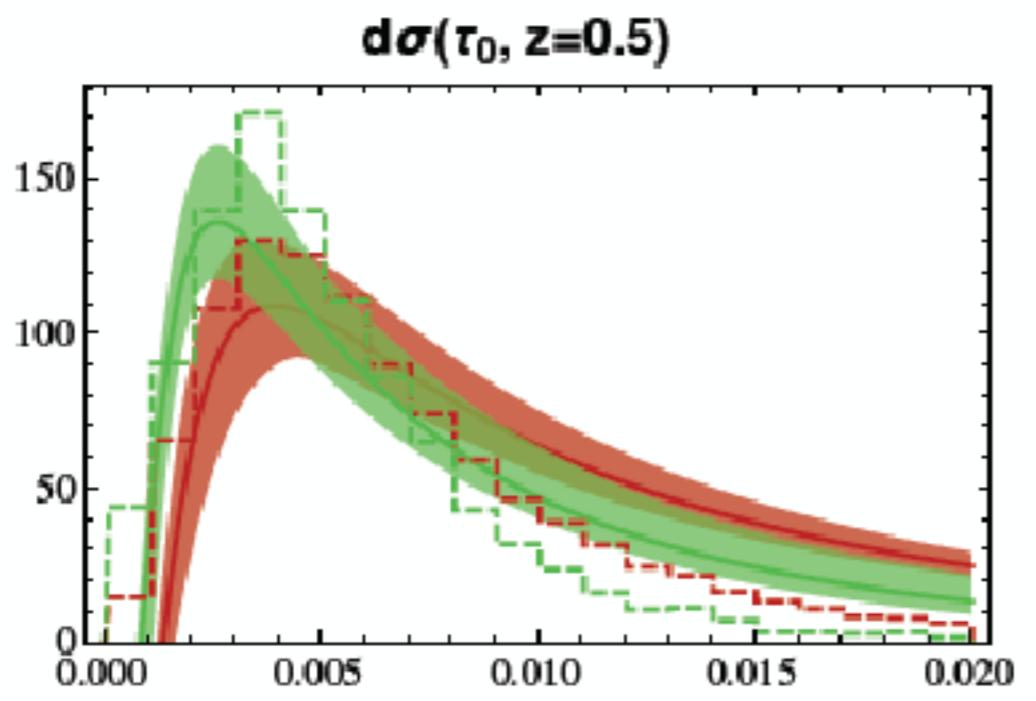
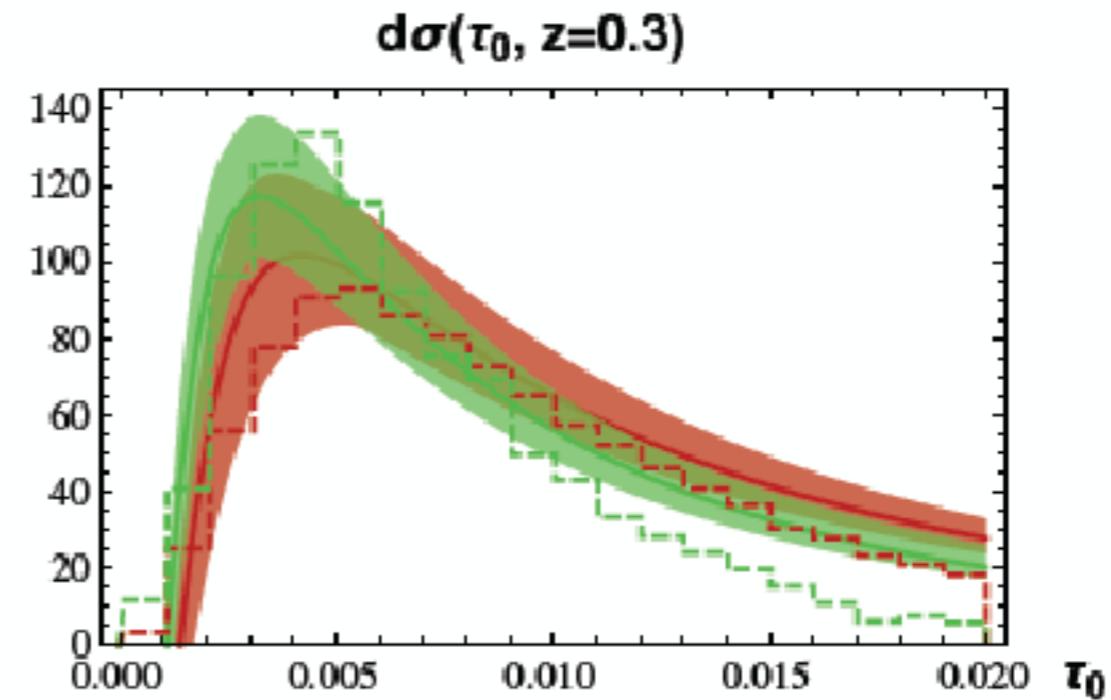
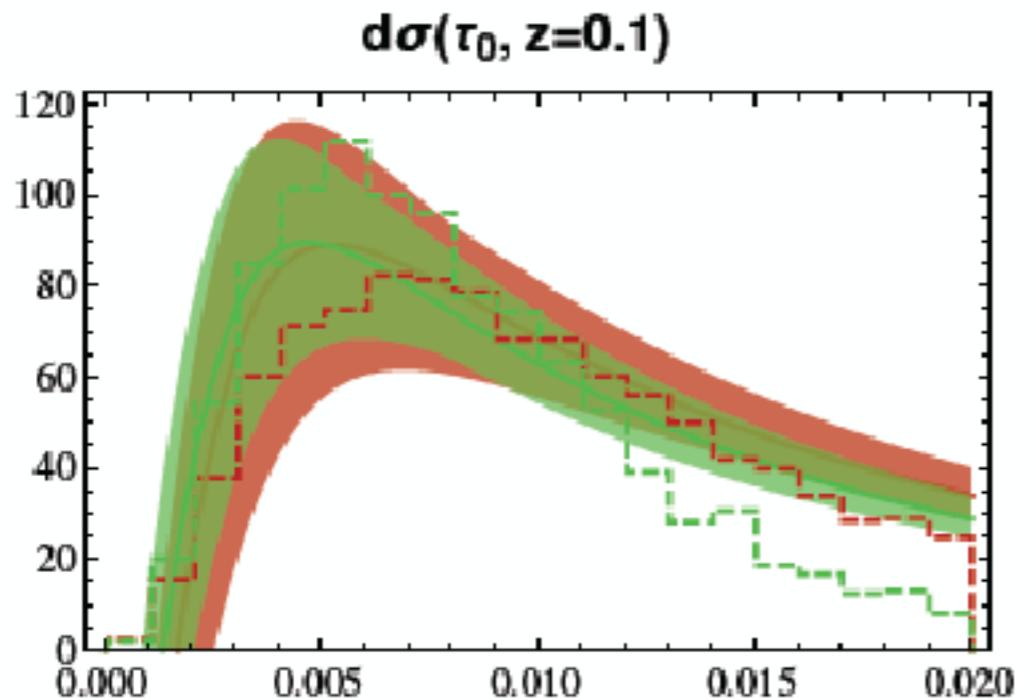


Force Madgraph to create  $J/\psi$  from gluon initiated jet

PYTHIA: parton shower, hadronization

# NLL' vs. Monte Carlo

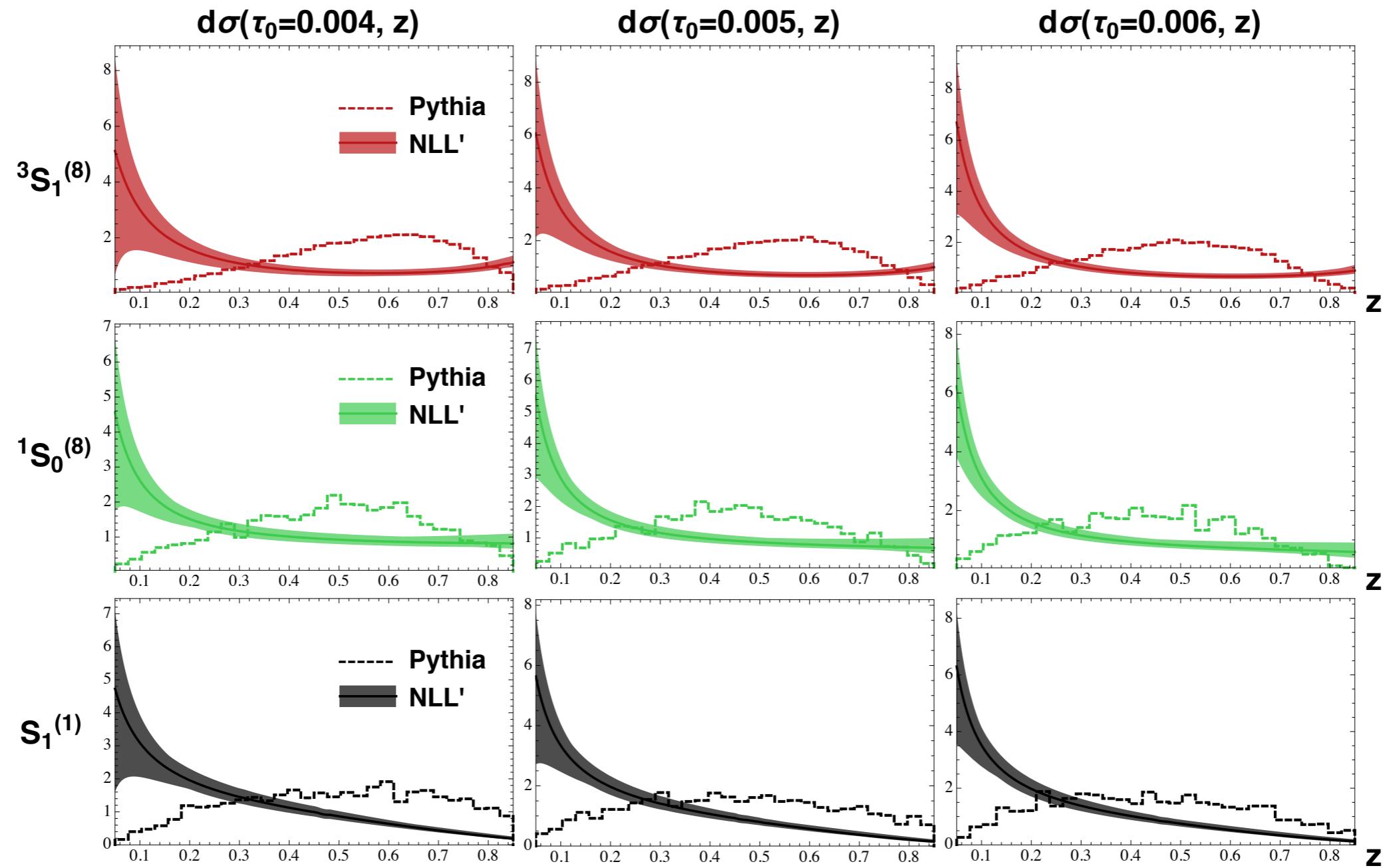
fixed  $z$ , variable  $\tau_0$



good agreement, some discrimination for large  $z$

# NLL' FJF vs. Pythia

R. Bain, L. Dai, A. Hornig, A. K. Leibovich, Y. Makris, T. Mehen JHEP 1606 (2016) 121



$e^+e^- \rightarrow \bar{q}qg$

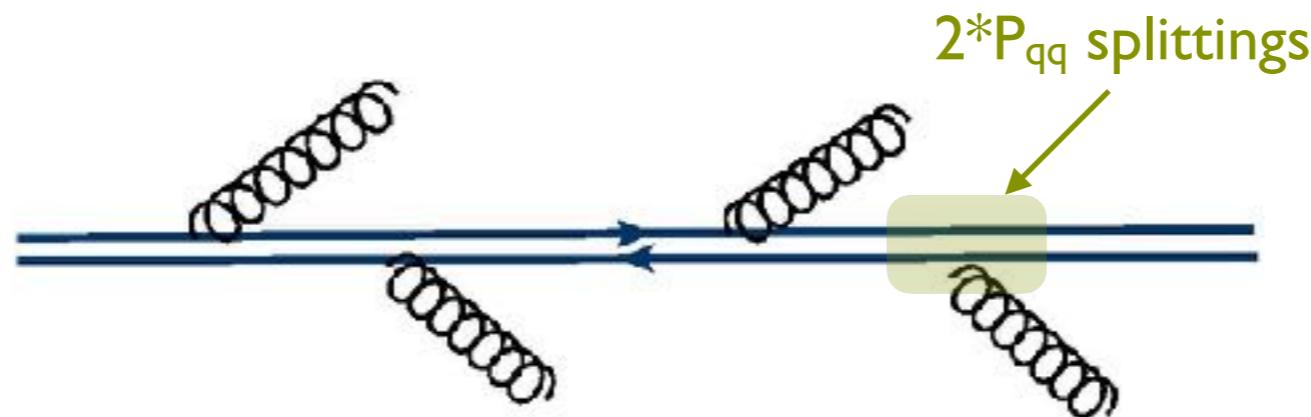
$E_{CM} = 250 \text{ GeV}$

$\tau_0 = s/\omega^2$

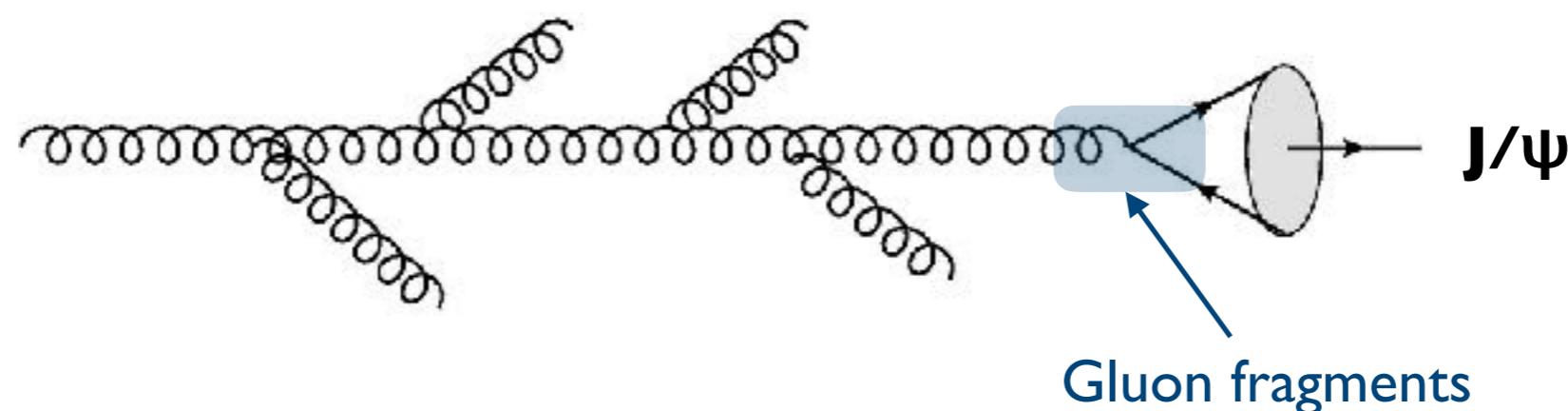
$\hookrightarrow \text{jet w/ } J/\Psi$

# Explaining difference between NLL' vs Pythia

PYTHIA's model for showering color-octet  $c\bar{c}$  pairs:



Physical picture of analytical calculation

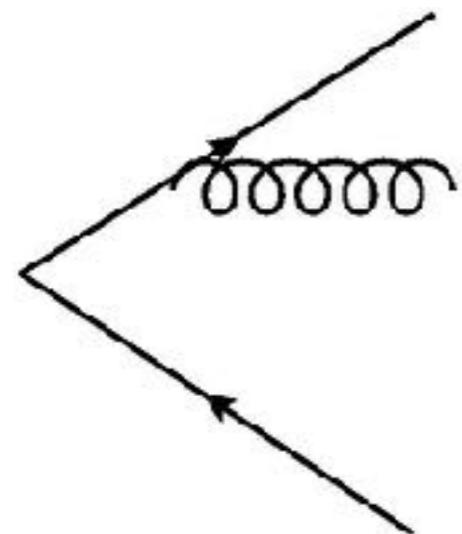


Pythia z distributions much harder than NLL' calculations

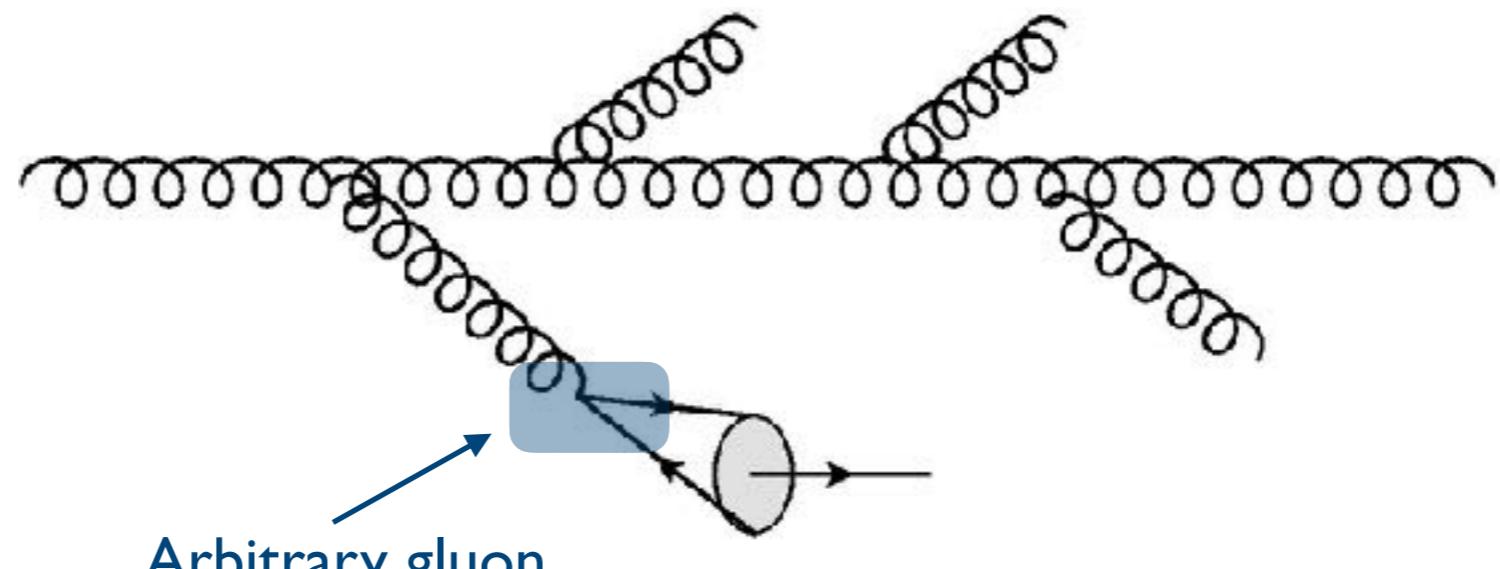
# Gluon Fragmentation Improved PYTHIA (GFIP)

**Madgraph 5**

$$e^+ e^- \rightarrow b \bar{b} g$$



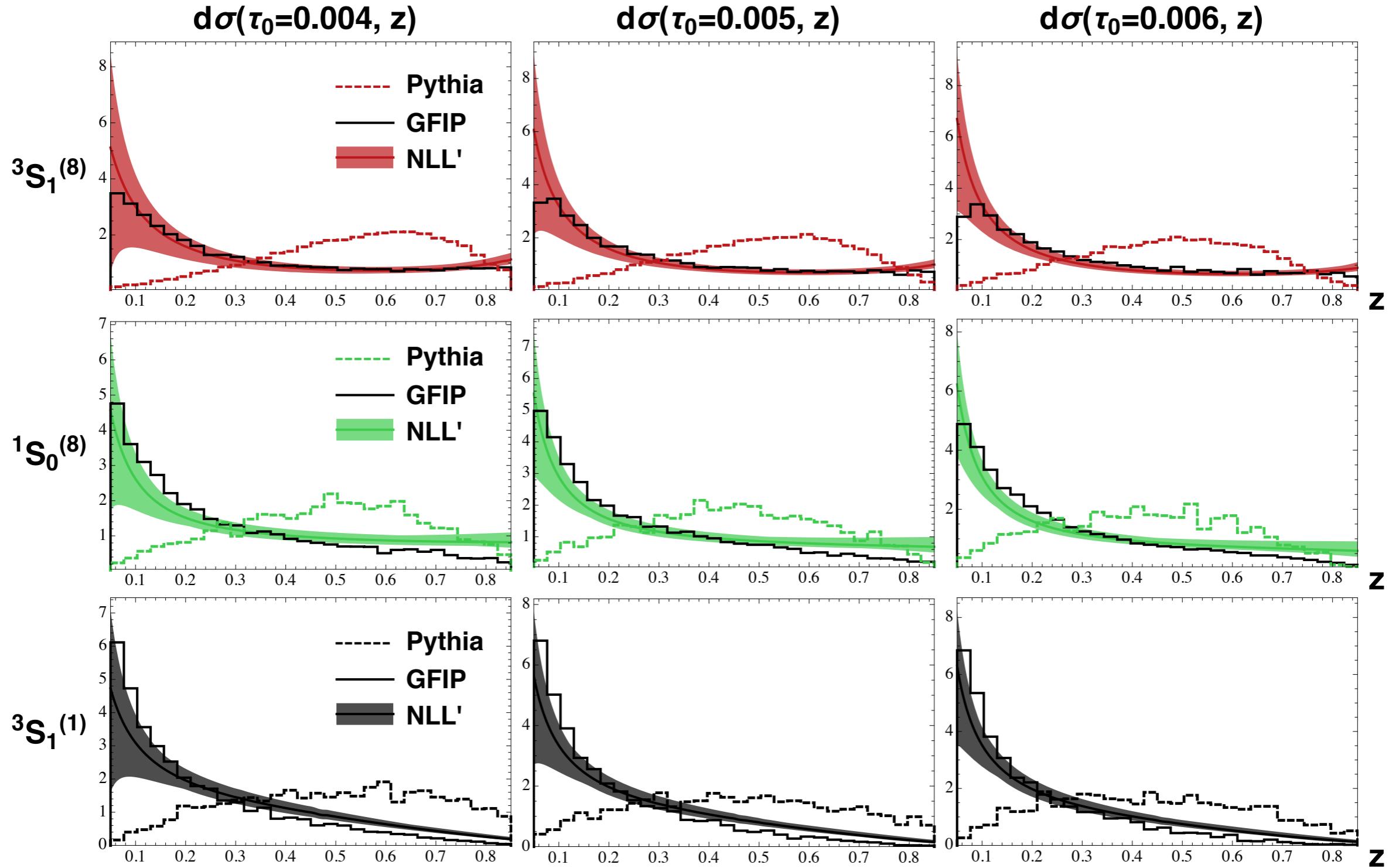
**PYTHIA + Convolution**



shower gluon with PYTHIA down to scale  $\sim 2m_c$ , no hadronization

convolve final state gluon distribution w/ NRQCD FFs

# NLL', PYTHIA, and GFIP

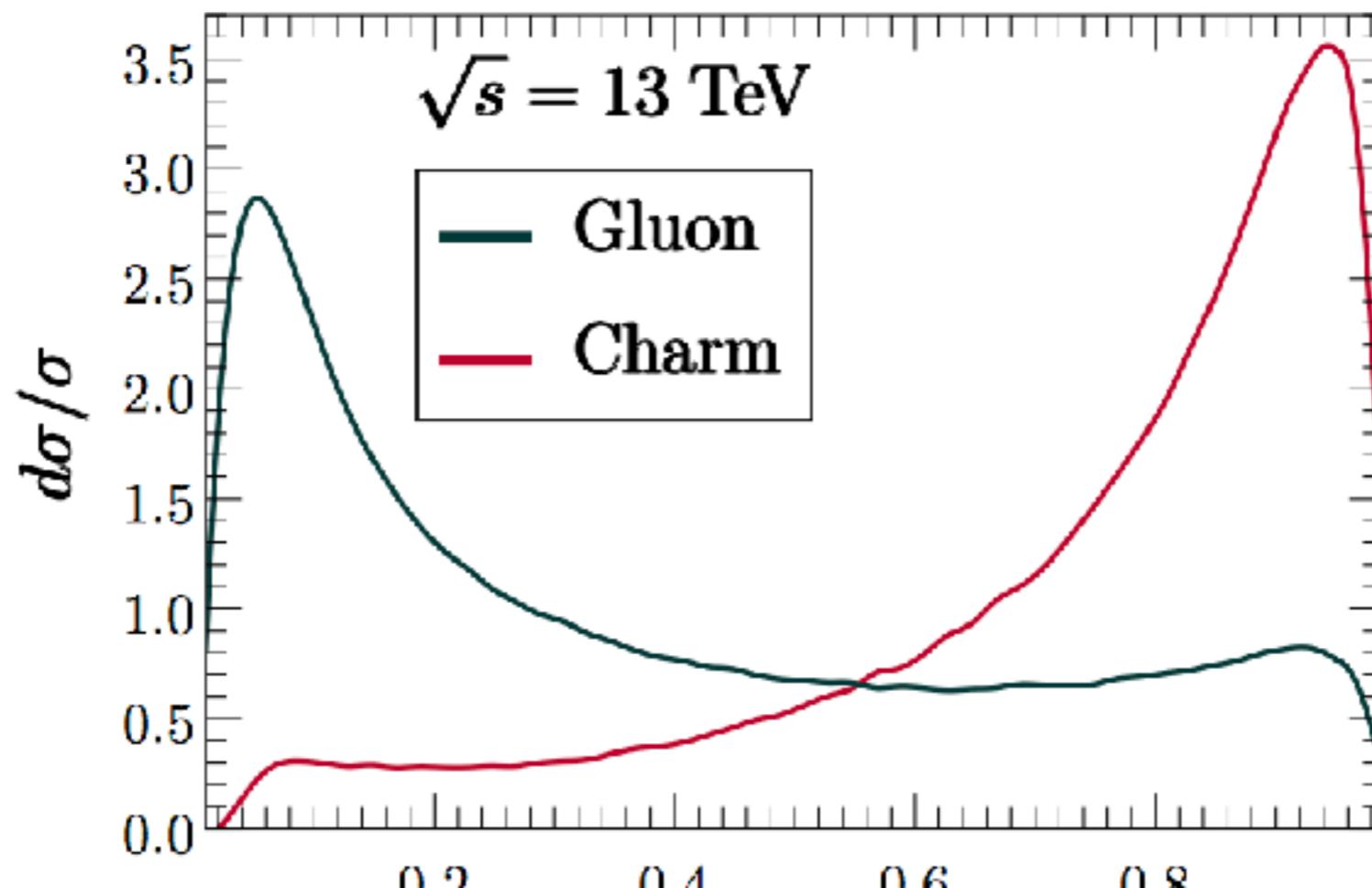


# GFIP and Recent LHCb Observations

R. Bain, L. Dai, A. K. Leibovich, Y. Makris, T. Mehen, accepted in PRL

generate events with hard c-quark , gluons

LHCb: pp collisions  $\sqrt{s} = 13 \text{ TeV}$  cuts:  $2 < \eta < 4.5$   
 $R = 0.5$   
 $p_{T,\text{JET}} < 20 \text{ GeV}$   
evolve shower to scale  $\sim 2m_c$   $p_\mu < 5 \text{ GeV}$



convolve w/ NRQCD FF for c quarks, gluons  $\sim 2m_c$

LHCb data is normalized so  $\sum_i \Delta z \left( \frac{d\sigma}{\sigma} \right)_i = \Delta z$

compare  $0.1 < z < 0.9$

Use following three sets of LDMEs

	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle$ $\times \text{GeV}^3$	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$ $\times 10^{-2} \text{ GeV}^3$	$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$ $\times 10^{-2} \text{ GeV}^3$	$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle / m_c^2$ $\times 10^{-2} \text{ GeV}^3$
B & K [5, 6]	$1.32 \pm 0.20$	$0.224 \pm 0.59$	$4.97 \pm 0.44$	$-0.72 \pm 0.88$
Chao, et al. [12]	$1.16 \pm 0.20$	$0.30 \pm 0.12$	$8.9 \pm 0.98$	$0.56 \pm 0.21$
Bodwin et al. [13]	$1.32 \pm 0.20$	$1.1 \pm 1.0$	$9.9 \pm 2.2$	$0.49 \pm 0.44$

Butenschoen and Kniehl, PRD 84 (2011) 051501

global fits to world's data

Chao, et. al. PRL 108, 242004 (2012)

fits to high  $p_T$  hadron collider data

Bodwin, et. al., PRL 113, 022001(2014)

# FJF and Recent LHCb Observations

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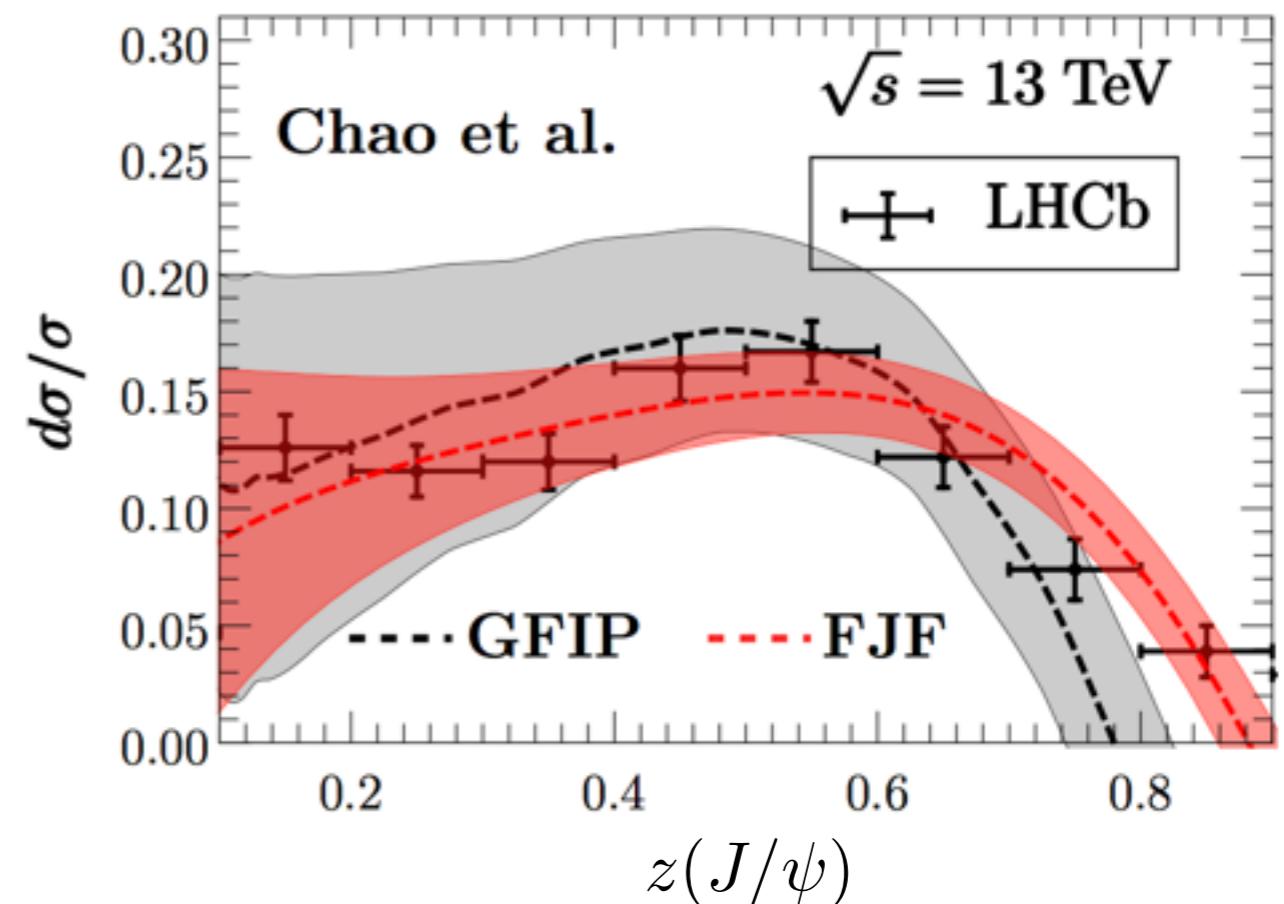
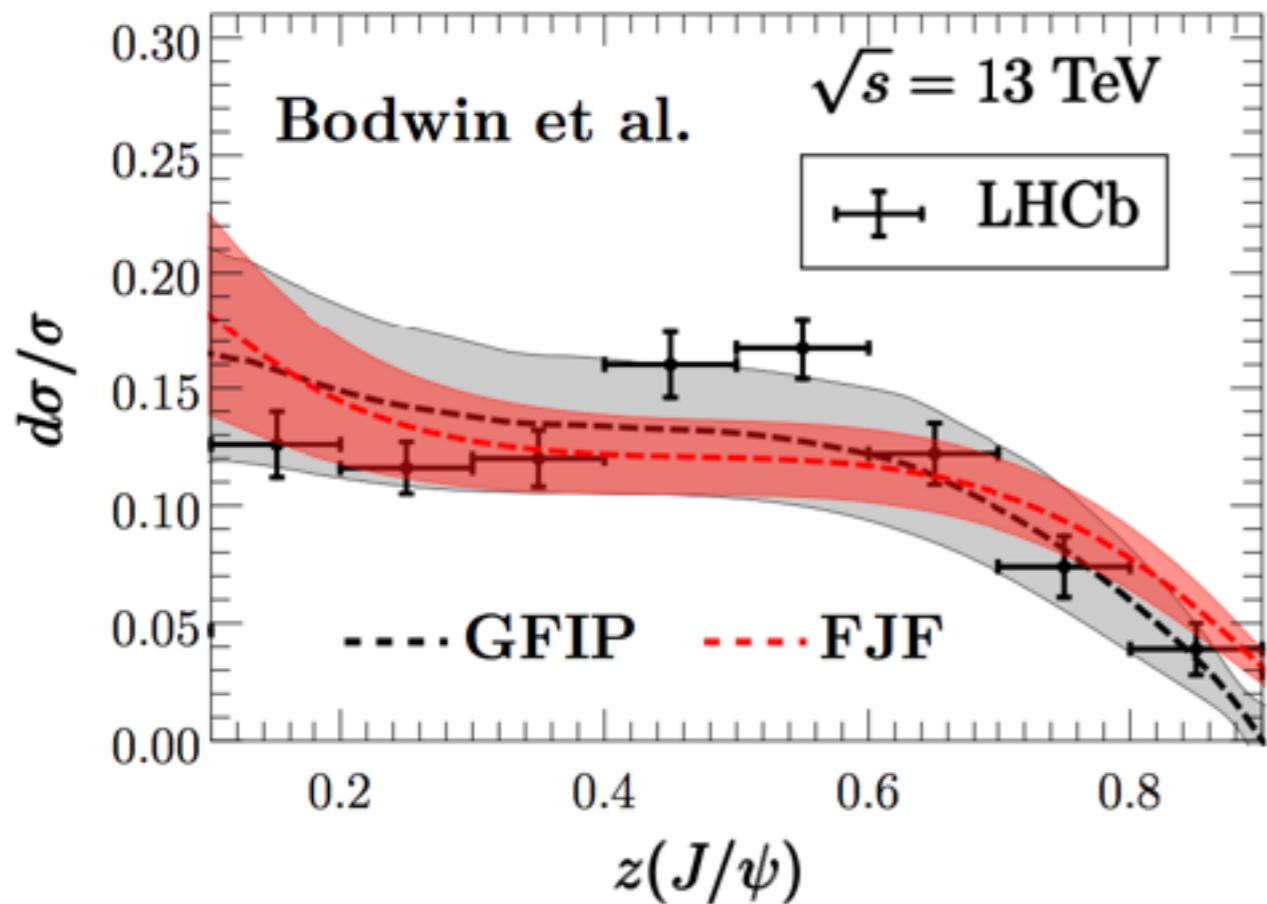
combine FJFs with hard events generated by Madgraph

NRQCD FFs evolved from  $2m_c$  to jet energy scale using DGLAP

factorization theorem with tree level hard function,  
trivial soft function, no NLL' resummation

FJF is only term in factorization dependent on  $z(J/\psi)$

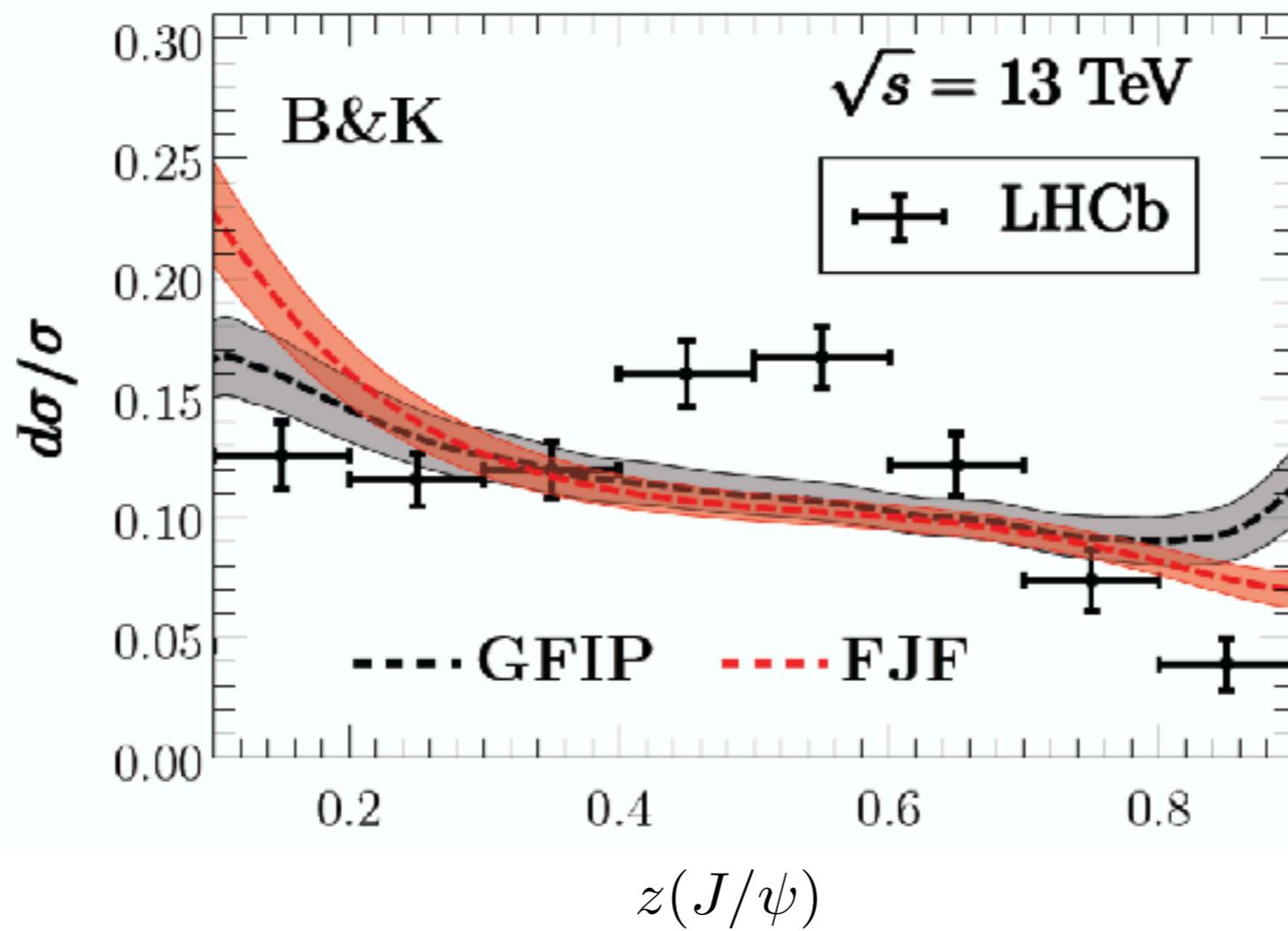
# Results



FJFs, GFIP consistent

LDME from fits high  $p_T$  agree with LHCb

# Results

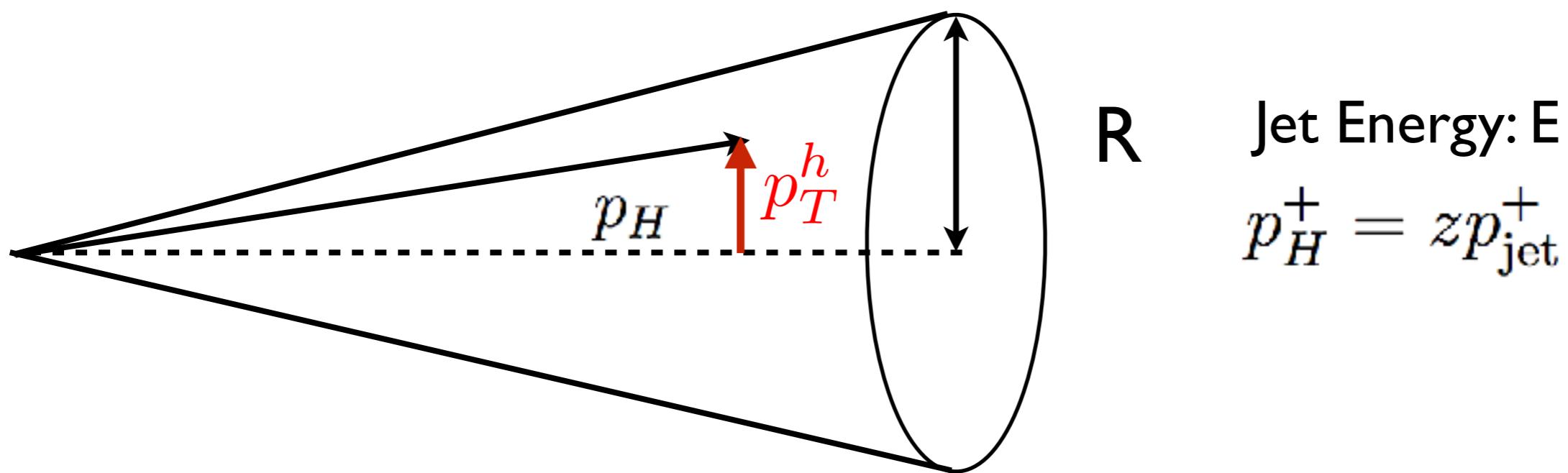


LDME from global fits:  
poorer agreement with LHCb, better than PYTHIA

# Transverse Momentum Dependent FJFs

R. Bain, Y. Makris, TM, JHEP 1611 (2016) 144

jets with identified hadron: hadron z,  $p_T$  are both measured

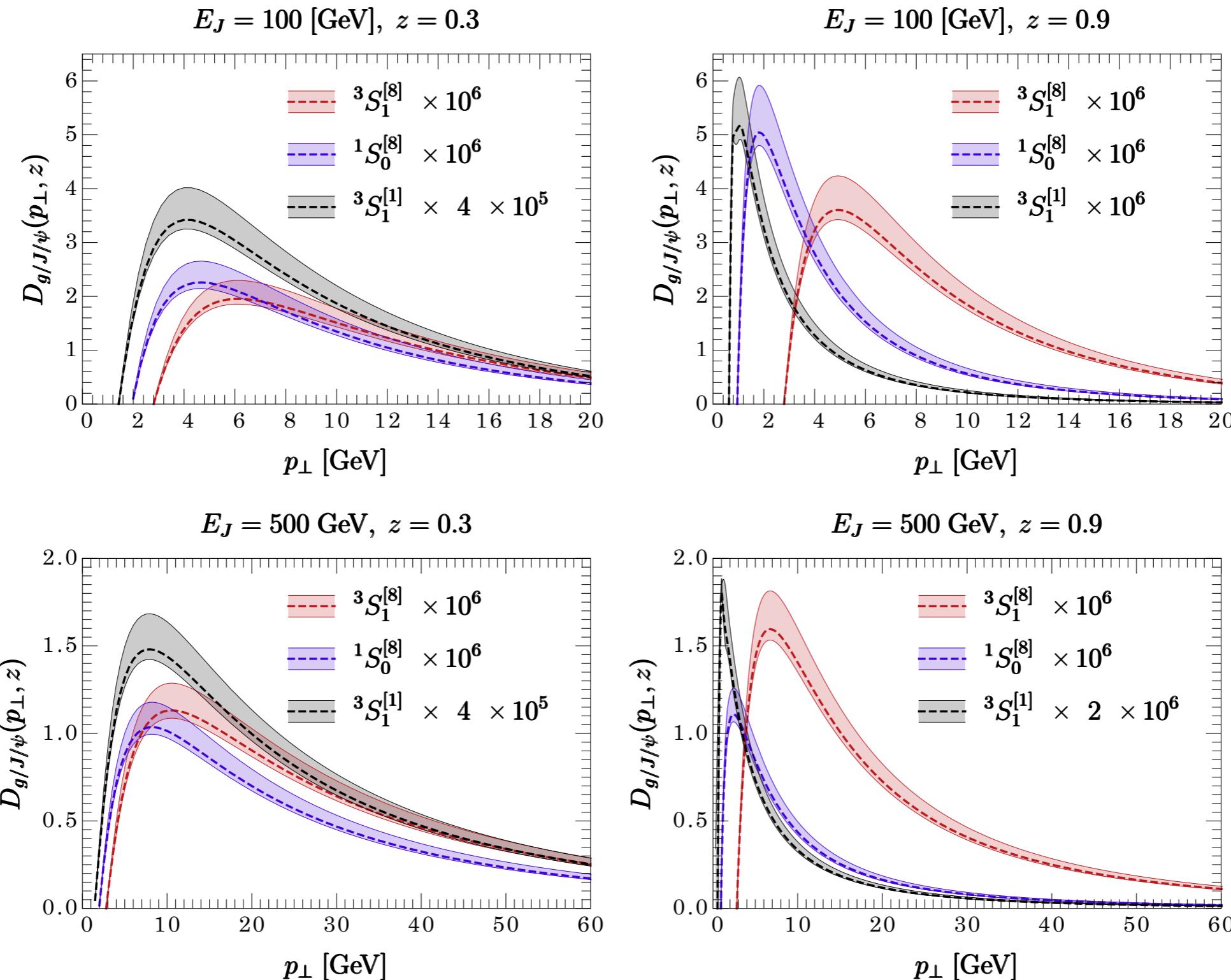


transverse momentum measured w/ rspt. to jet axis

jet axis  $\sim$  parton initiating jet if out of jet radiation is ultrasoft

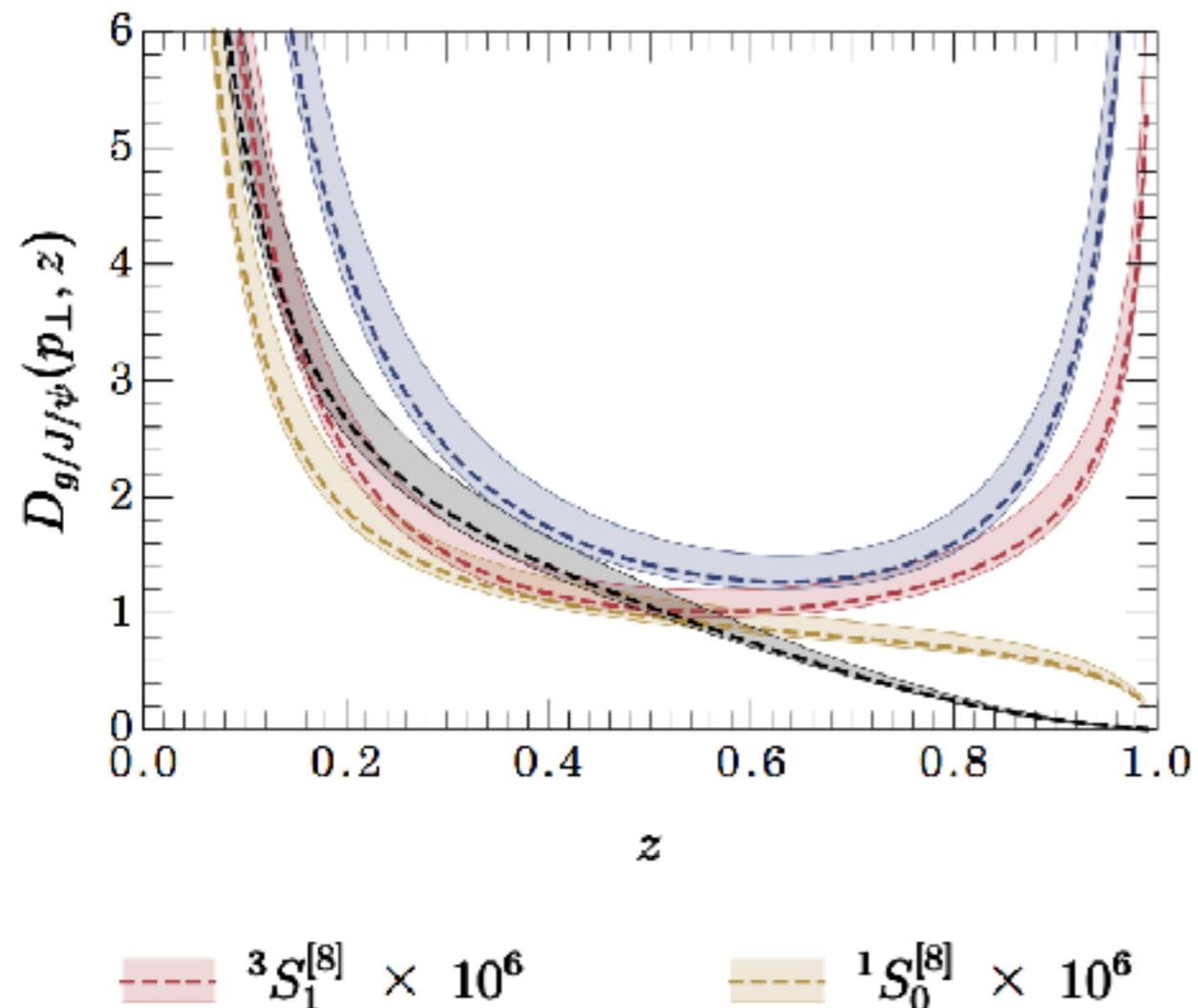
$$\omega \gg p_T^h \gg \Lambda \gg \Lambda_{\text{QCD}}$$

# Application to Quarkonium Production

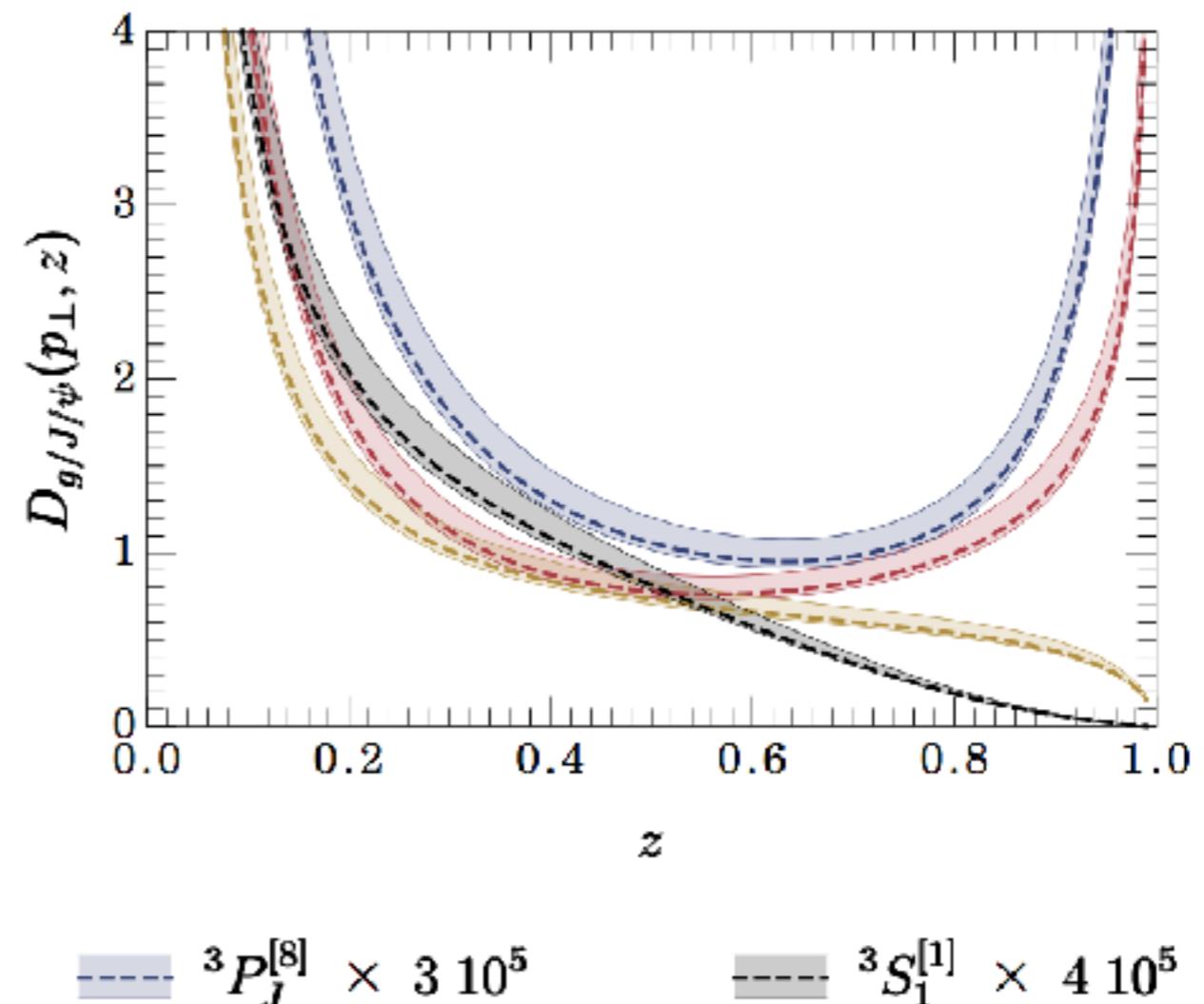


# Application to Quarkonium Production

$E_J = 100 \text{ GeV}, p_\perp = 10 \text{ GeV}$

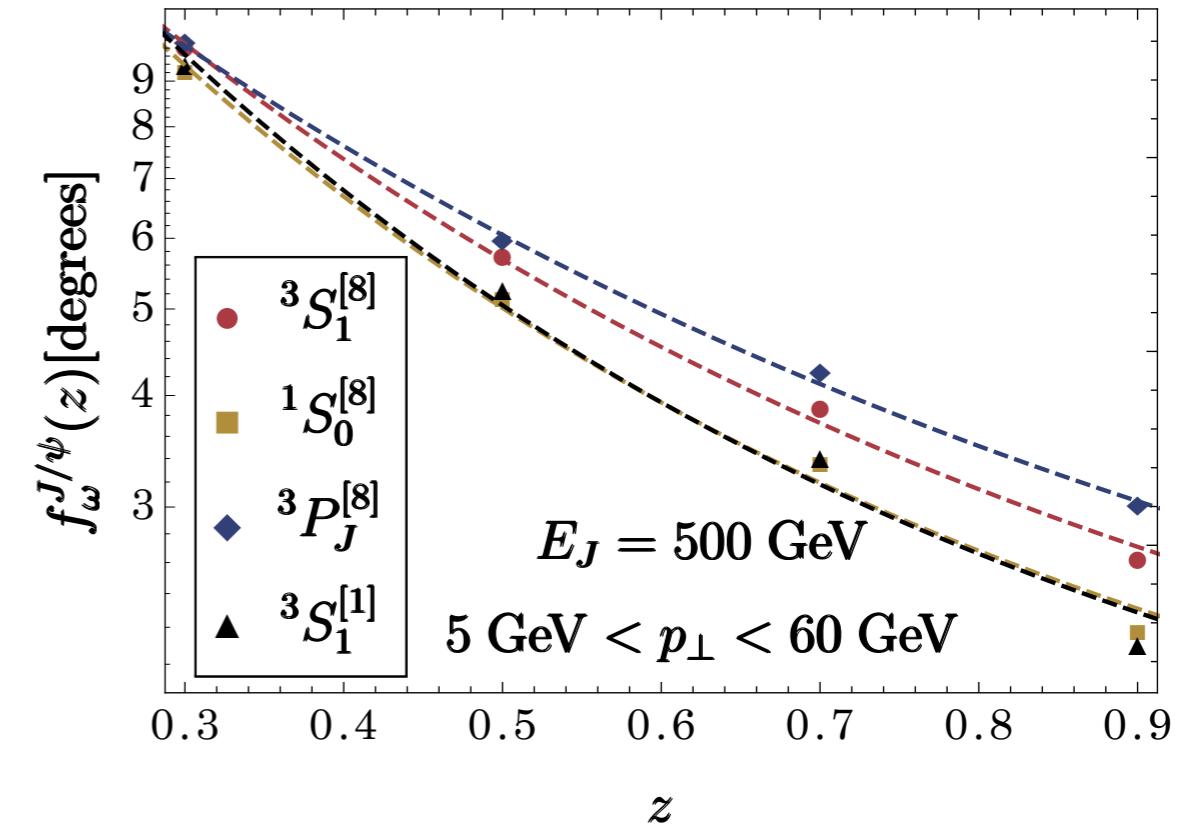
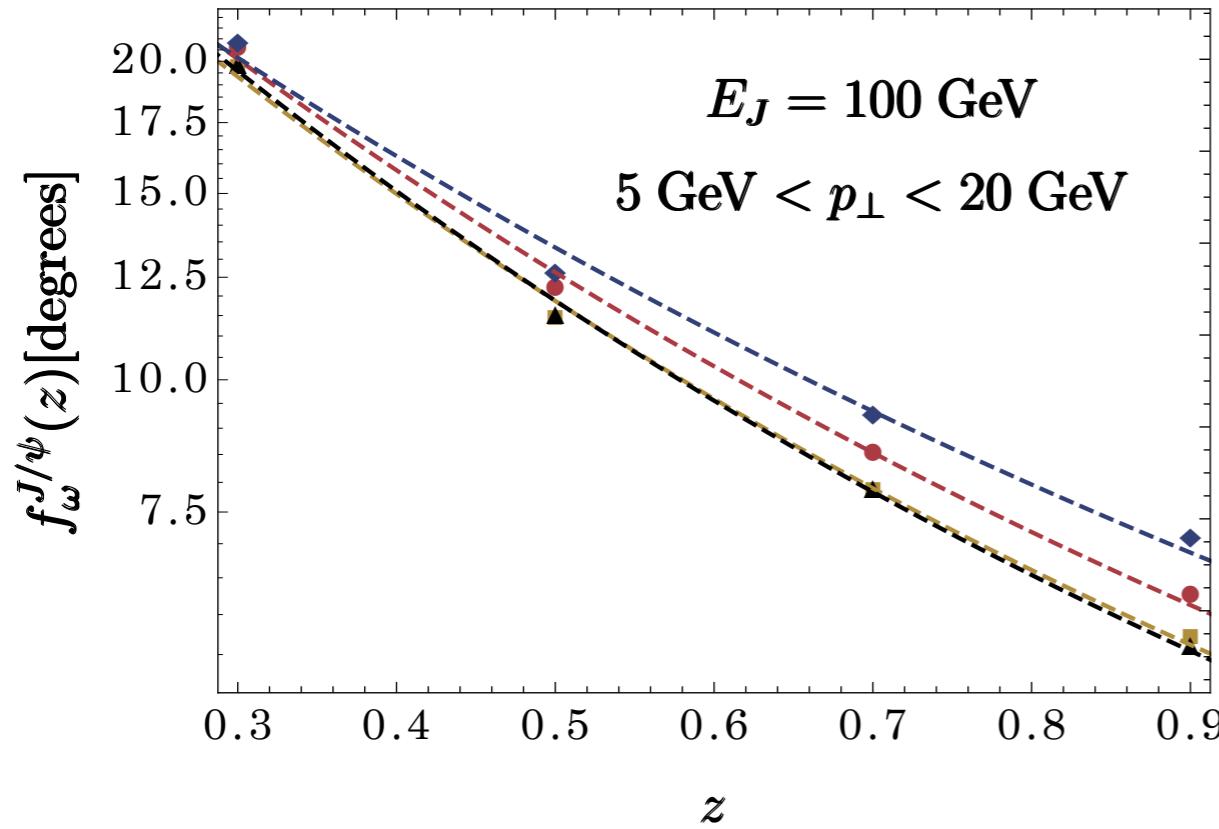


$E_J = 500 \text{ GeV}, p_\perp = 10 \text{ GeV}$



# Application to Quarkonium Production

$$\langle \theta \rangle(z) \sim \frac{2 \int dp_\perp p_\perp D_{g/h}(p_\perp, z, \mu)}{z\omega \int dp_\perp D_{g/h}(p_\perp, z, \mu)} \equiv f_\omega^h(z)$$



$E_J = 100 \text{ GeV}$		
${}^{2S+1}L_J^{[1,8]}$	$C_0$	$C_1$
${}^3S_1^{[1]}$	3.92	0.92
${}^3S_1^{[8]}$	3.86	0.84
${}^1S_0^{[8]}$	3.88	0.90
${}^3P_J^{[8]}$	3.75	0.74

$E_J = 500 \text{ GeV}$		
${}^{2S+1}L_J^{[1,8]}$	$C_0$	$C_1$
${}^3S_1^{[1]}$	3.75	1.68
${}^3S_1^{[8]}$	3.48	1.39
${}^1S_0^{[8]}$	3.66	1.64
${}^3P_J^{[8]}$	3.28	1.20

$$\ln(f(x)) = g(x; C_0, C_1) \text{ s.t. } g(x=0) = C_0$$

$$g_2(x) = C_0 \exp(-C_1 x)$$

## Conclusions

measuring quarkonia within jets and using jet observables  
should provide insights into quarkonia production

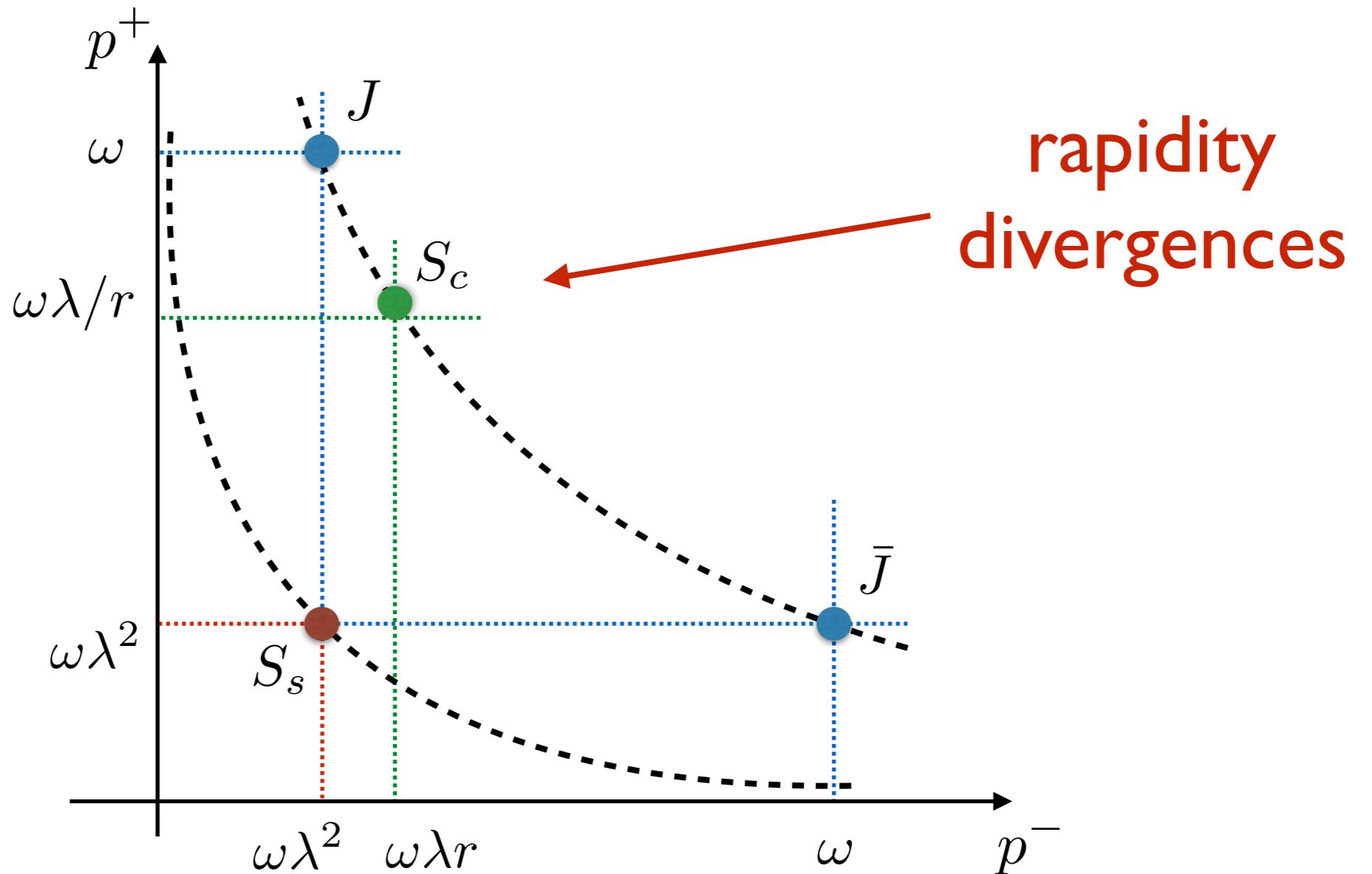
If  ${}^1S_0^{(8)}$  mechanism dominates high  $p_T$  production  
FJF should have negative slope for  $z(E)$ , for  $z > 0.5$

LHCb data on  $z(J/\psi)$  well-described by FJF, GFIP  
improvement over default PYTHIA, consistent w/ NLL' calculations  
LDME extracted from high  $p_T$  slightly preferred

TMD FJFs:  $p_T^h, \theta$  discriminate between NRQCD mechanisms

# Backup

# Scales in TMDFJF



$$p_c \sim \omega(\lambda^2, 1, \lambda) \quad p_{cs} \sim p_h^\perp(r, 1/r, 1) \quad p_{us} \sim \Lambda(1, 1, 1)$$

$$\lambda = p_h^\perp / \omega$$

# Factorization Theorem

$$D_{q/h}(\mathbf{p}_\perp,z,\mu) = H_+(\mu) \times \left[ \mathcal{D}_{q/h} \otimes_\perp S_C \right](\mathbf{p}_\perp,z,\mu)$$

$$H_+(\mu)=(2\pi)^2N_c\;C_+^\dagger(\mu)C_+(\mu)$$

$$\begin{aligned}\mathcal{D}_{q/h}(\mathbf{p}_\perp^{\mathcal{D}},z)\equiv&\frac{1}{z}\sum_{X_n}\frac{1}{2N_c}\delta(p^-_{Xh;r})\delta^{(2)}(p^\perp_{Xh;r})\operatorname{Tr}\Big[\frac{\not{n}}{2}\langle 0|\delta_{\omega,\overline{\mathcal{P}}}\chi_n(0)\delta^{(2)}(\mathcal{P}_\perp^{X_n}+\mathbf{p}_\perp^{\mathcal{D}})|X_nh\rangle\\&\quad\times\langle X_nh|\bar{\chi}_n(0)|0\rangle\Big]\end{aligned}$$

$$\mathcal{D}_{i/h}(\mathbf{p}_\perp,z,\mu,\nu)=\int_z^1\frac{dx}{x}\;\mathcal{J}_{i/j}(\mathbf{p}_\perp,x,\mu,\nu)D_{j/h}\left(\frac{z}{x},\mu\right)\;+\;\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{|\mathbf{p}_\perp|^2}\right)$$

$$S_C(\mathbf{p}_\perp^S)\equiv\frac{1}{N_c}\sum_{X_{cs}}\operatorname{Tr}\Big[\langle 0|V_n^\dagger(0)U_n(0)\delta^{(2)}(\mathcal{P}_\perp+\mathbf{p}_\perp^S)|X_{cs}\rangle\langle X_{cs}|U_n^\dagger(0)V_n(0)|0\rangle\Big]$$

# Anomalous Dimensions for RGE, RRGE

## RGE

$$\gamma_\mu^{S_C}(\nu) = \frac{\alpha_s C_i}{\pi} \ln \left( \frac{\mu^2}{r^2 \nu^2} \right)$$

$$\gamma_\mu^D(\nu) + \gamma_\mu^{S_C}(\nu) = \gamma_\mu^J = \frac{\alpha_s C_i}{\pi} \left( \ln \left( \frac{\mu^2}{r^2 \omega^2} \right) + \bar{\gamma}_i \right)$$

$$\gamma_\mu^D(\nu) = \frac{\alpha_s C_i}{\pi} \left( \ln \left( \frac{\nu^2}{\omega^2} \right) + \bar{\gamma}_i \right)$$

## Rapidity Renormalization Group

$$\gamma_\nu^{S_C}(p_\perp, \mu) = +(8\pi)\alpha_s C_i \mathcal{L}_0(\mathbf{p}_\perp, \mu^2)$$

$$\gamma_\nu^D(\mathbf{p}_\perp, \mu) + \gamma_\nu^S(\mathbf{p}_\perp, \mu) = 0$$

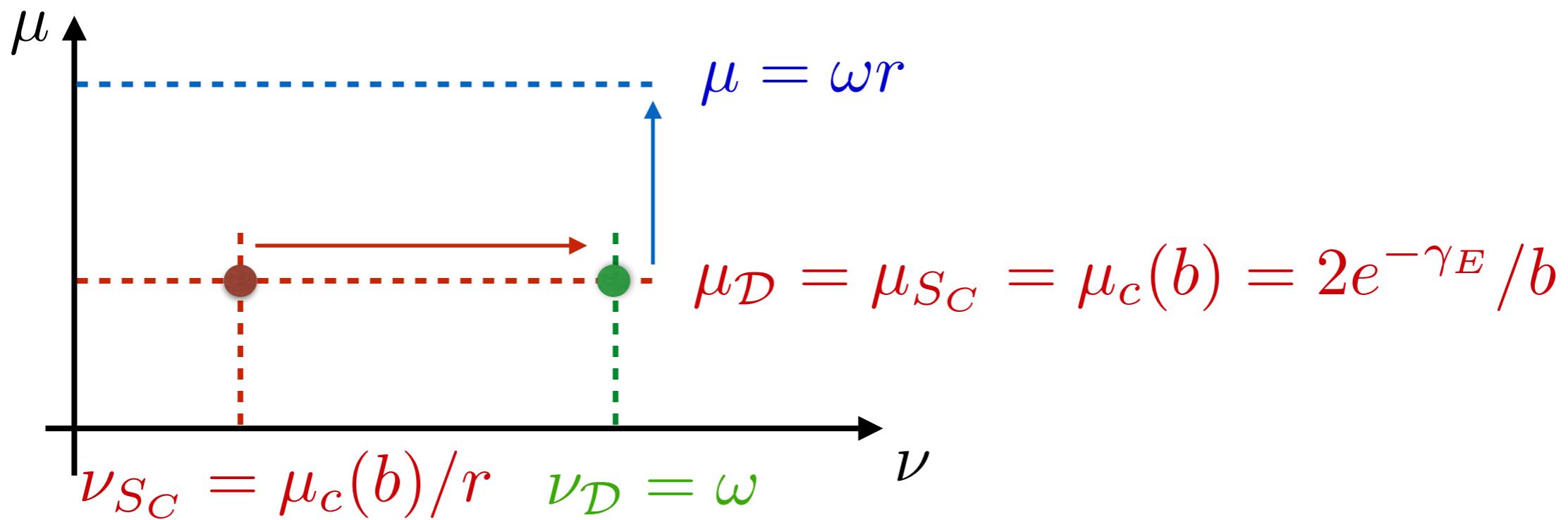
$$\gamma_\nu^D(p_\perp, \mu) = -(8\pi)\alpha_s C_i \mathcal{L}_0(\mathbf{p}_\perp, \mu^2)$$

J.-y. Chiu, A. Jain, D. Neill, I.Z. Rothstein, PRL 108 (2012) 151601

J.-y. Chiu, A. Jain, D. Neill, I.Z. Rothstein, JHEP 1205 (2012) 084

# Solution to Evolution Eqs. in Fourier Space

$$D_{i/h}(p_\perp, z, \mu) = (2\pi)^2 p_\perp \int_0^\infty db b J_0(bp_\perp) \mathcal{U}_{S_C}(\mu, \mu_{S_C}, m_{S_C}) \mathcal{U}_{\mathcal{D}}(\mu, \mu_{\mathcal{D}}, 1)$$
$$\times \mathcal{V}_{S_C}(b, \mu_{S_C}, \nu_{\mathcal{D}}, \nu_{S_C}) \mathcal{FT} \left[ \mathcal{D}_{i/h}(\mathbf{p}_\perp, z, \mu_{\mathcal{D}}, \nu_{\mathcal{D}}) \otimes_{\perp} S_C^i(\mathbf{p}_\perp, \mu_{S_C}, \nu_{S_C}) \right]$$



fragmentation function (QCD)

$$D_q^h(z) = z \int \frac{dx^+}{4\pi} e^{ik^-x^+/2} \frac{1}{4N_c} \text{Tr} \sum_X \langle 0 | \bar{\eta} \Psi(x^+, 0, 0_\perp) | X h \rangle \langle X h | \bar{\Psi}(0) | 0 \rangle \Big|_{p_h^\perp=0}$$

fragmentation function (SCET)

$$D_q^h\left(\frac{p_h^-}{\omega}, \mu\right) = \pi\omega \int dp_h^+ \frac{1}{4N_c} \text{Tr} \sum_X \bar{\eta} \langle 0 | [\delta_{\omega, \bar{P}} \delta_{0, P_\perp} \chi_n(0)] | X h \rangle \langle X h | \bar{\chi}_n(0) | 0 \rangle$$

Jet function (SCET)

$$J_u(k^+\omega) = -\frac{1}{\pi\omega} \text{Im} \int d^4x e^{ik\cdot x} i \langle 0 | T \bar{\chi}_{n,\omega,0_\perp}(0) \frac{\bar{\eta}}{4N_c} \chi_n(x) | 0 \rangle$$

fragmentation jet function (SCET)

$$\mathcal{G}_{q,\text{bare}}^h(s, z) = \int d^4y e^{ik^+y^-/2} \int dp_h^+ \sum_X \frac{1}{4N_c} \text{tr} \left[ \frac{\bar{\eta}}{2} \langle 0 | [\delta_{\omega, \bar{P}} \delta_{0, P_\perp} \chi_n(y)] | X h \rangle \langle X h | \bar{\chi}_n(0) | 0 \rangle \right]$$

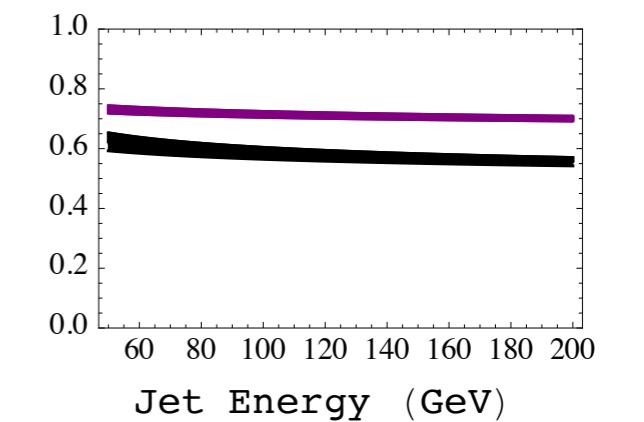
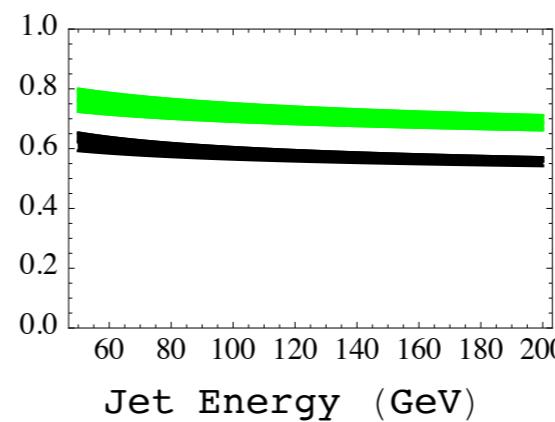
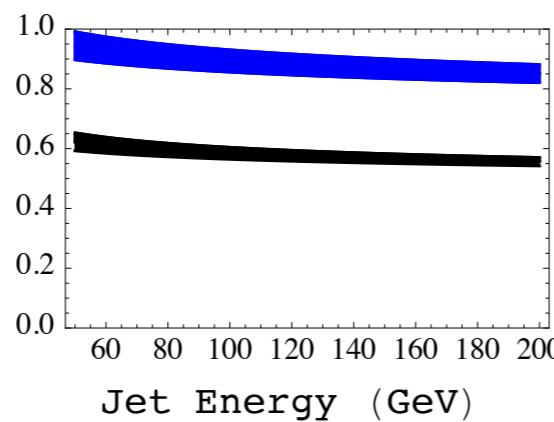
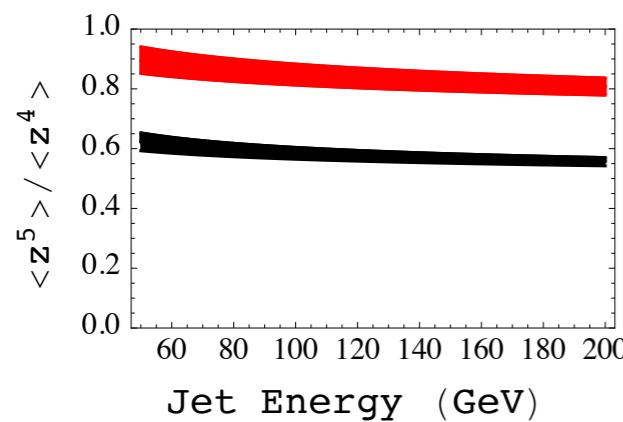
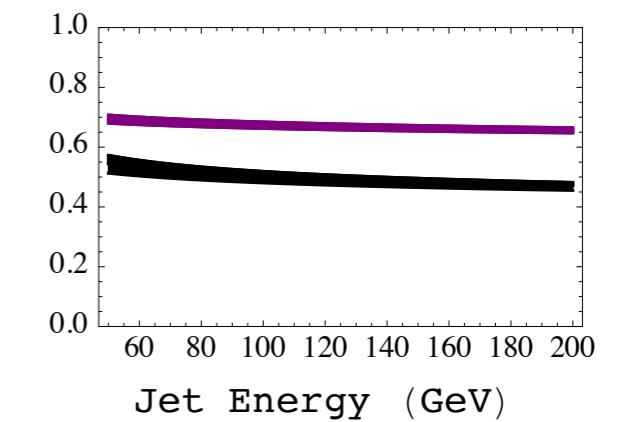
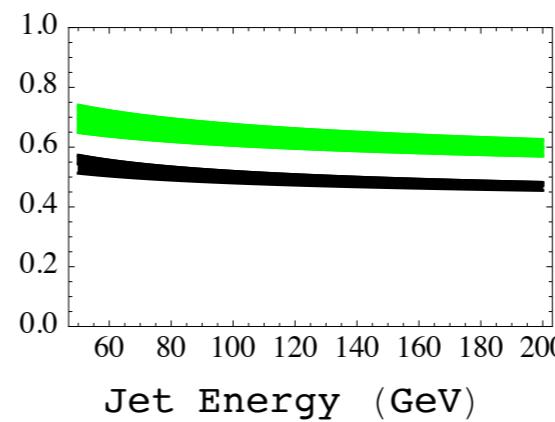
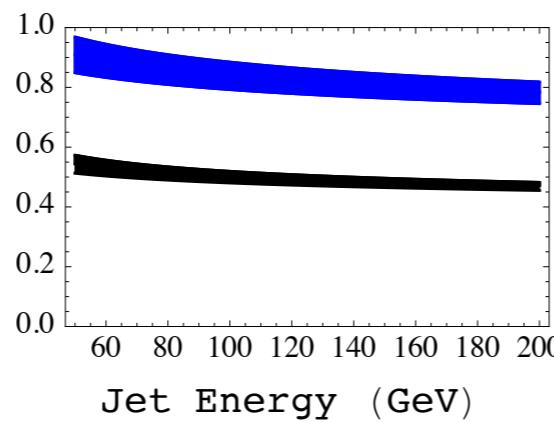
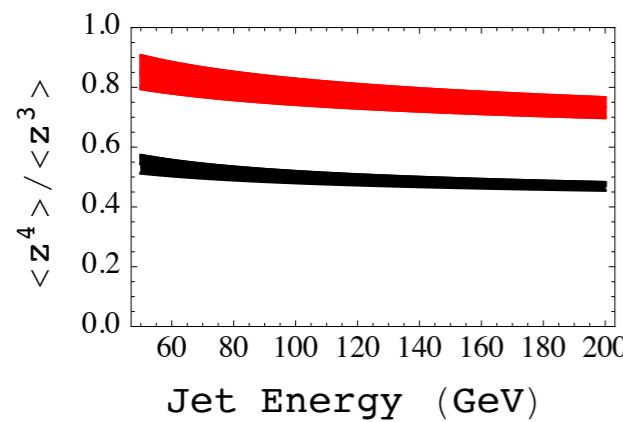
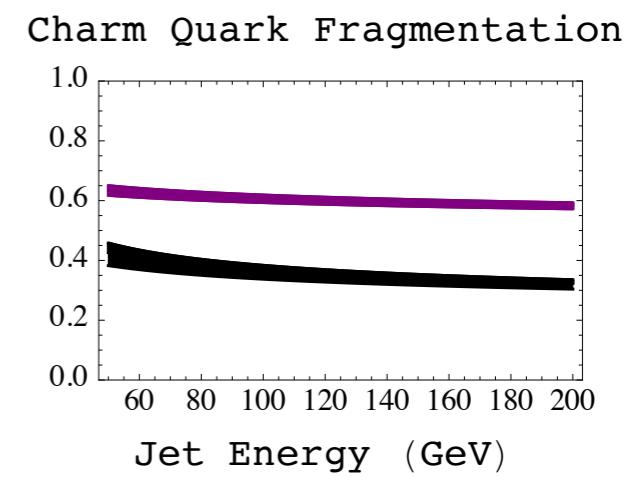
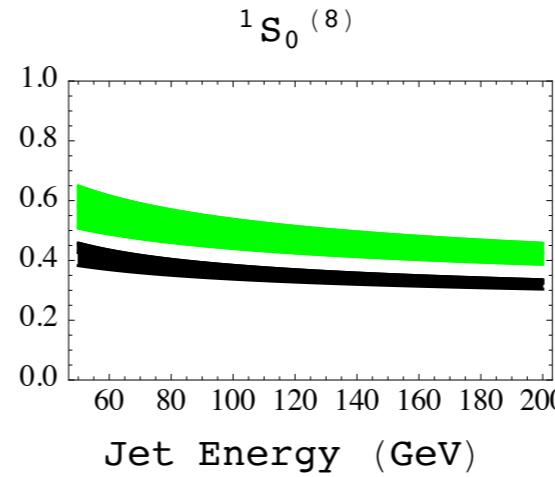
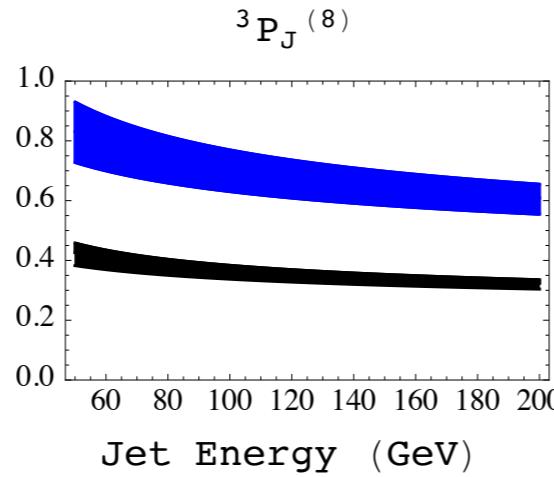
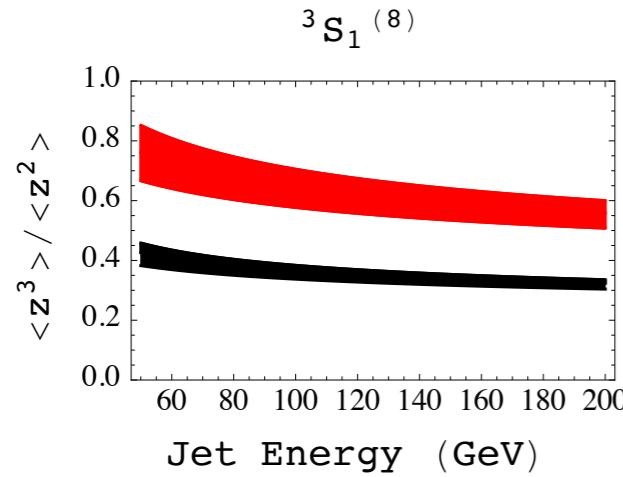
$$\delta(p^+/z - P_H^+) \rightarrow \delta(p^+/z - P_H^+) \delta(p^- - s/p^+)$$

FF

FJF

# Ratios of Moments

$$E \tan(R/2) < \mu < 4E \tan(R/2)$$

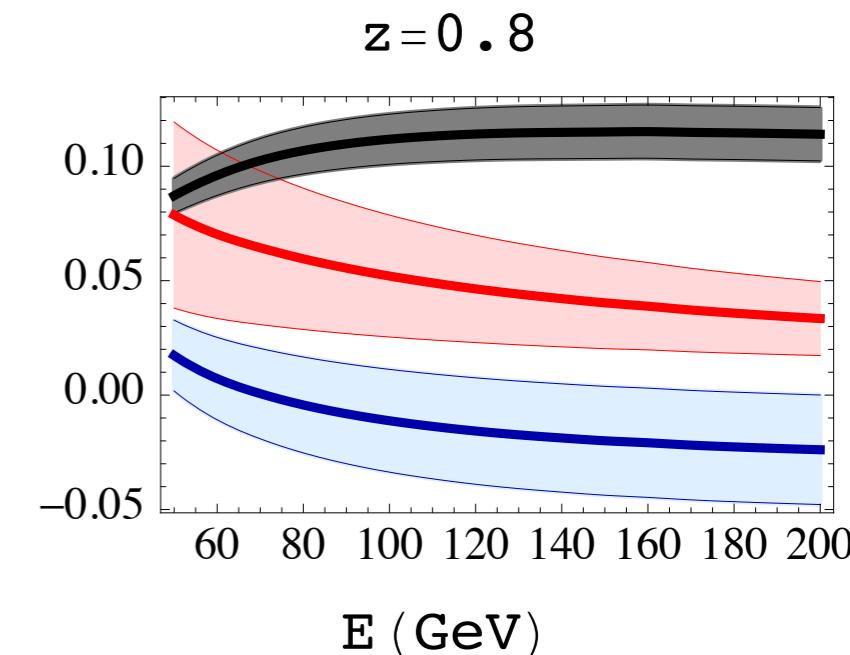
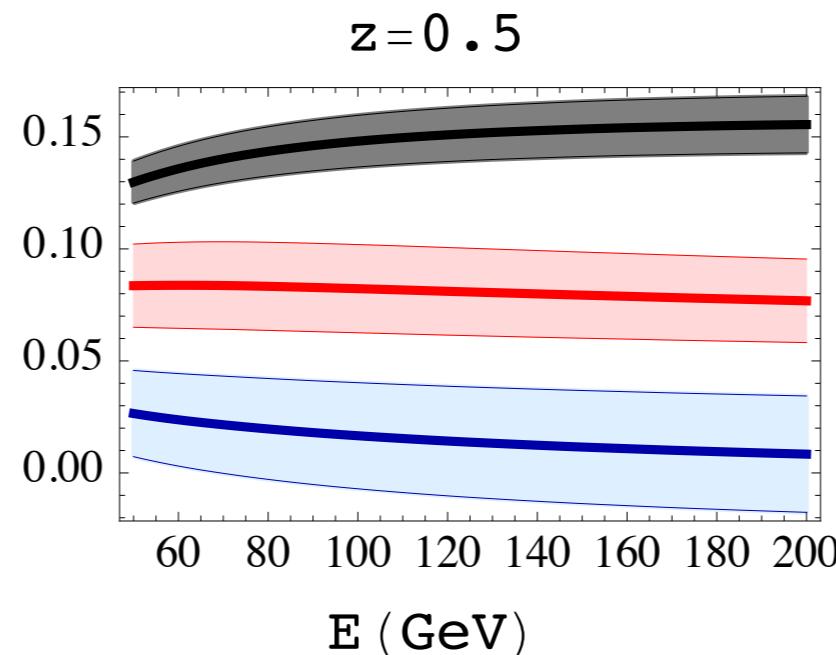
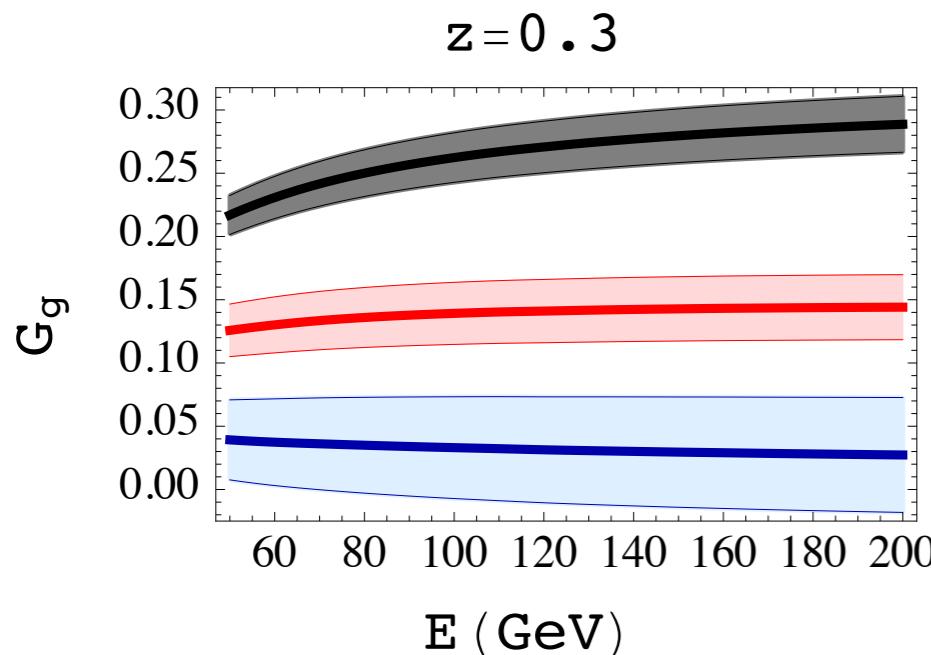


## Ratios of Moments

$$\frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{^3P_J^{(8)}} \approx \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{^3S_1^{(8)}} > \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{^1S_0^{(8)}} \approx \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{\text{c-quark}} > \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{^3S_1^{(1)}}$$

# Gluon FJF for different extractions of LDME

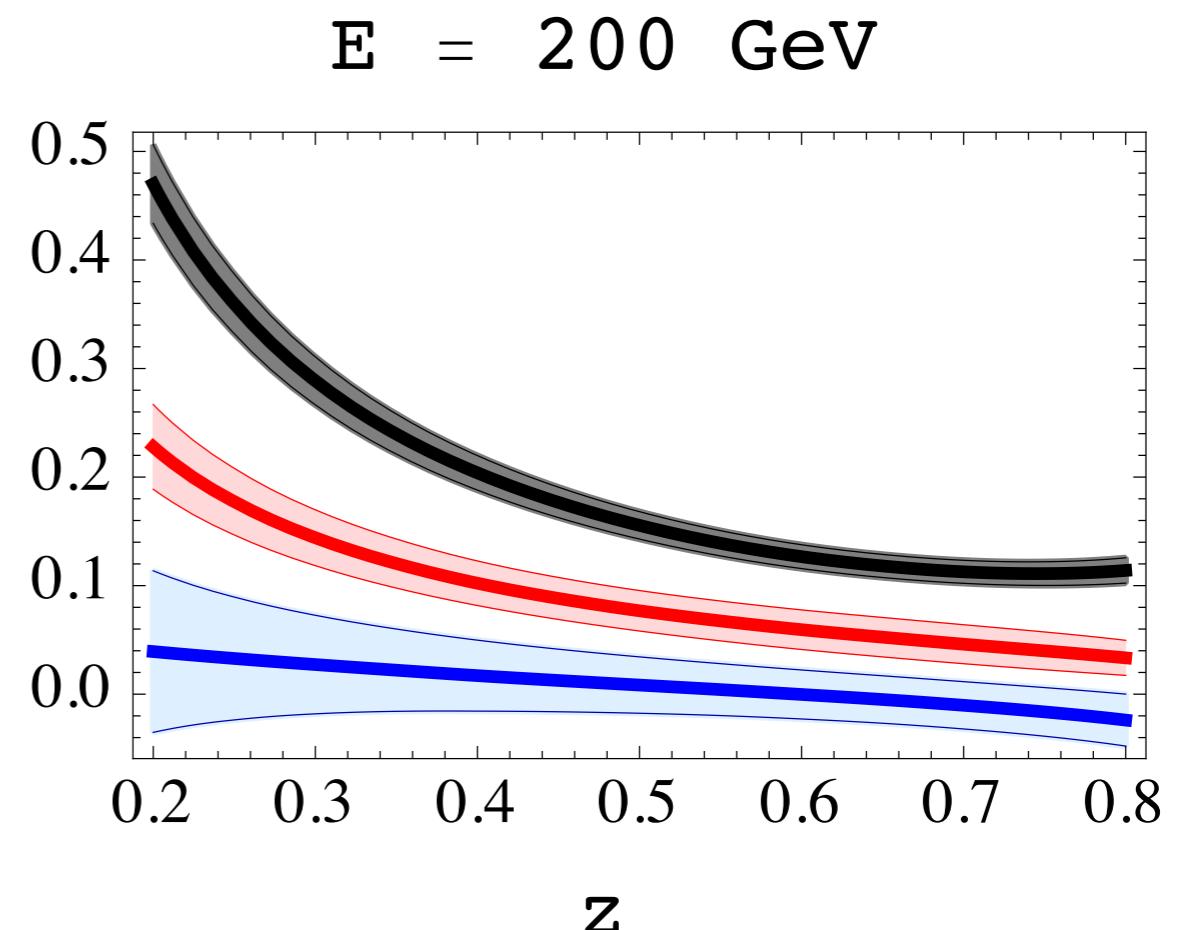
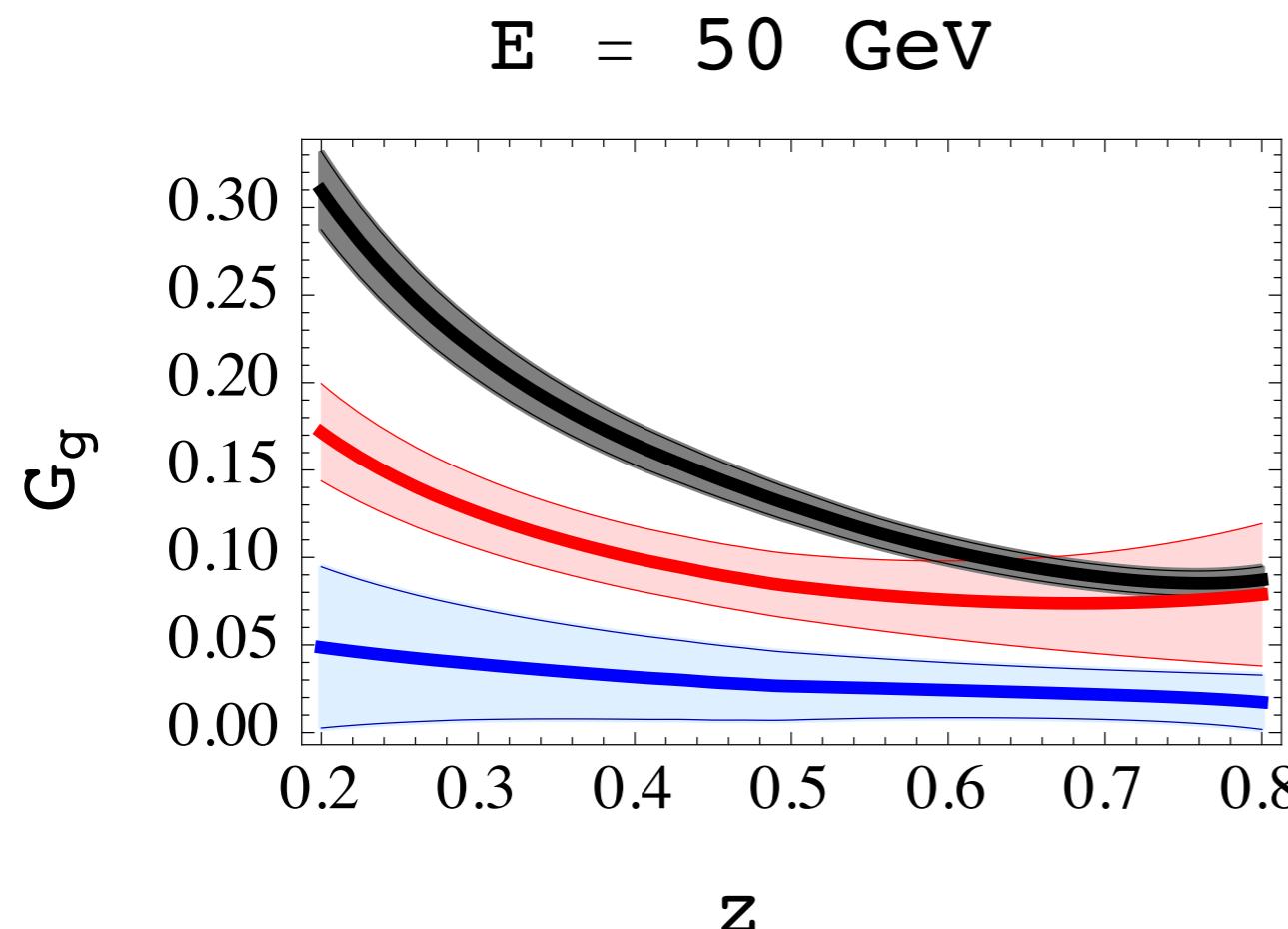
fix z, vary energy



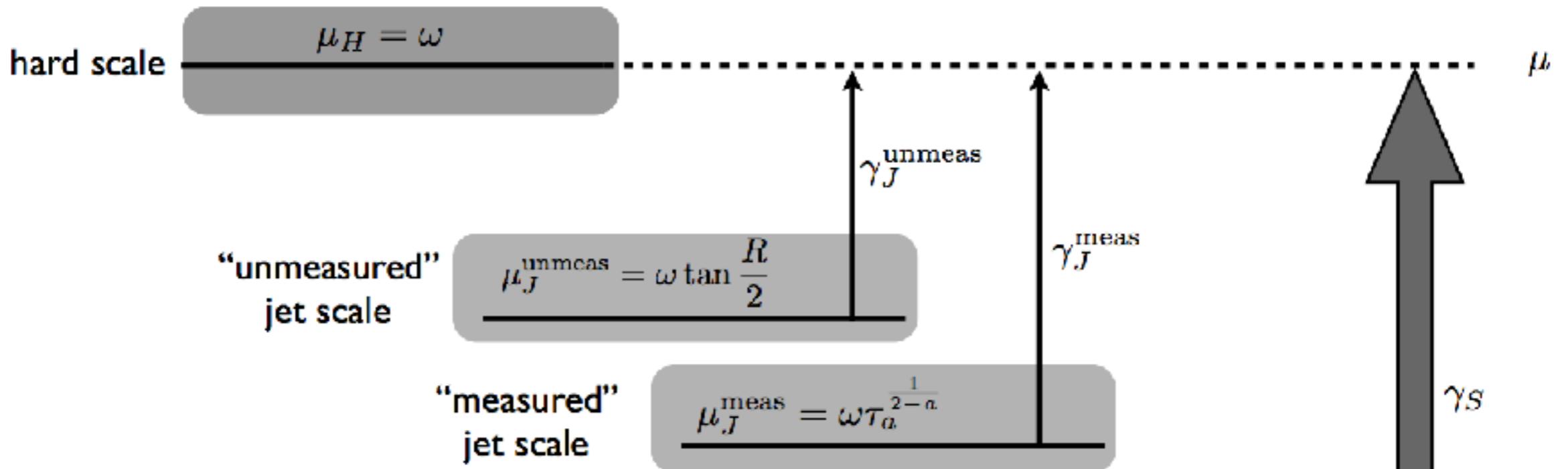
- Butenschoen and Kniehl, PRD 84 (2011) 051501, arXiv:1105.0822
- Bodwin, et. al. arXiv:1403.3612
- Chao, et. al. PRL 108, 242004 (2012)

# Gluon FJF for different extractions of LDME

fix energy, vary z



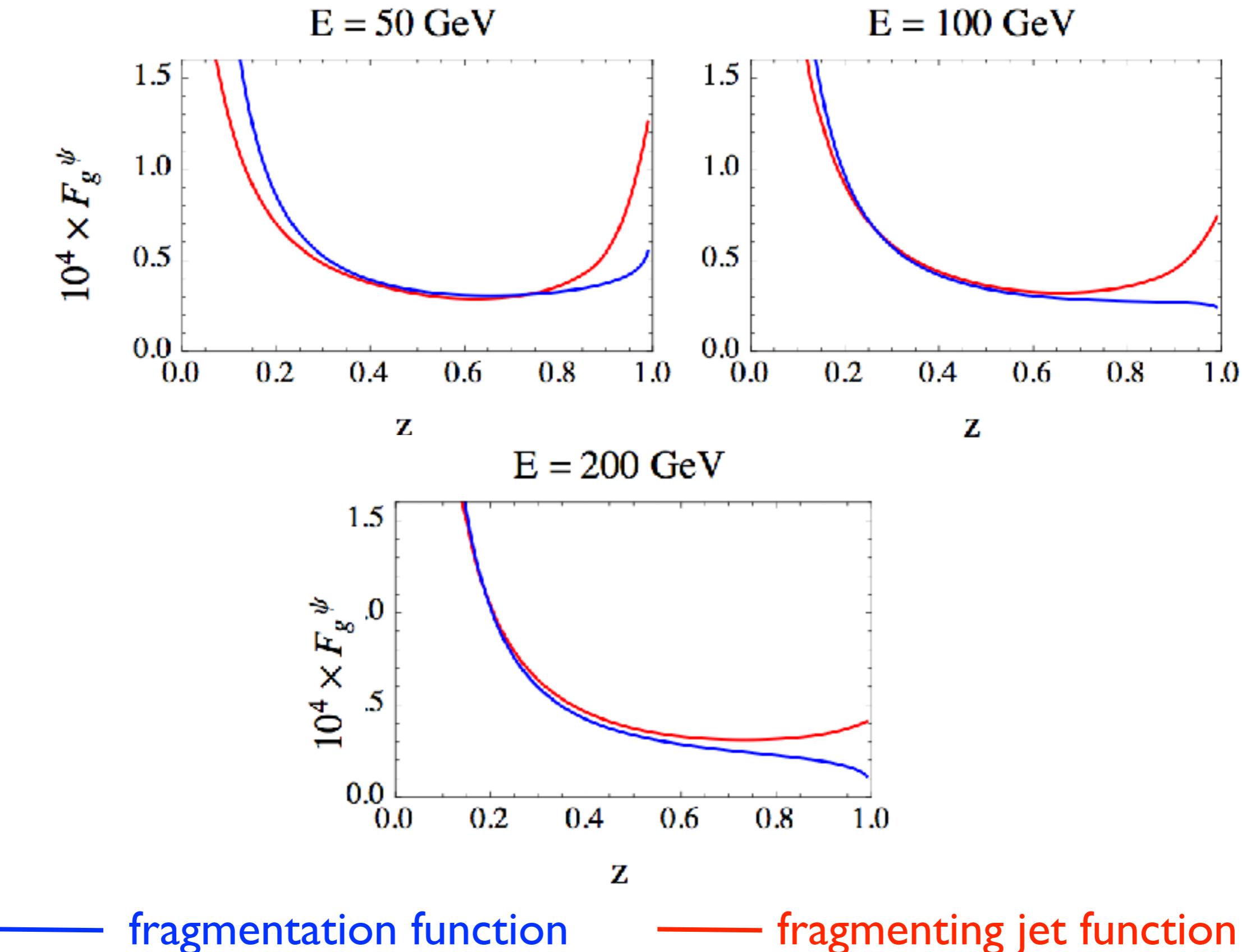
# Scales in Jet Cross section



EFT counting	matching/ matrix element	$\Gamma_{\text{cusp}}$	$\gamma_{H,J,S}$	$\beta[\alpha_s]$
LL	tree	1-loop	tree	1-loop
NLL	tree	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop

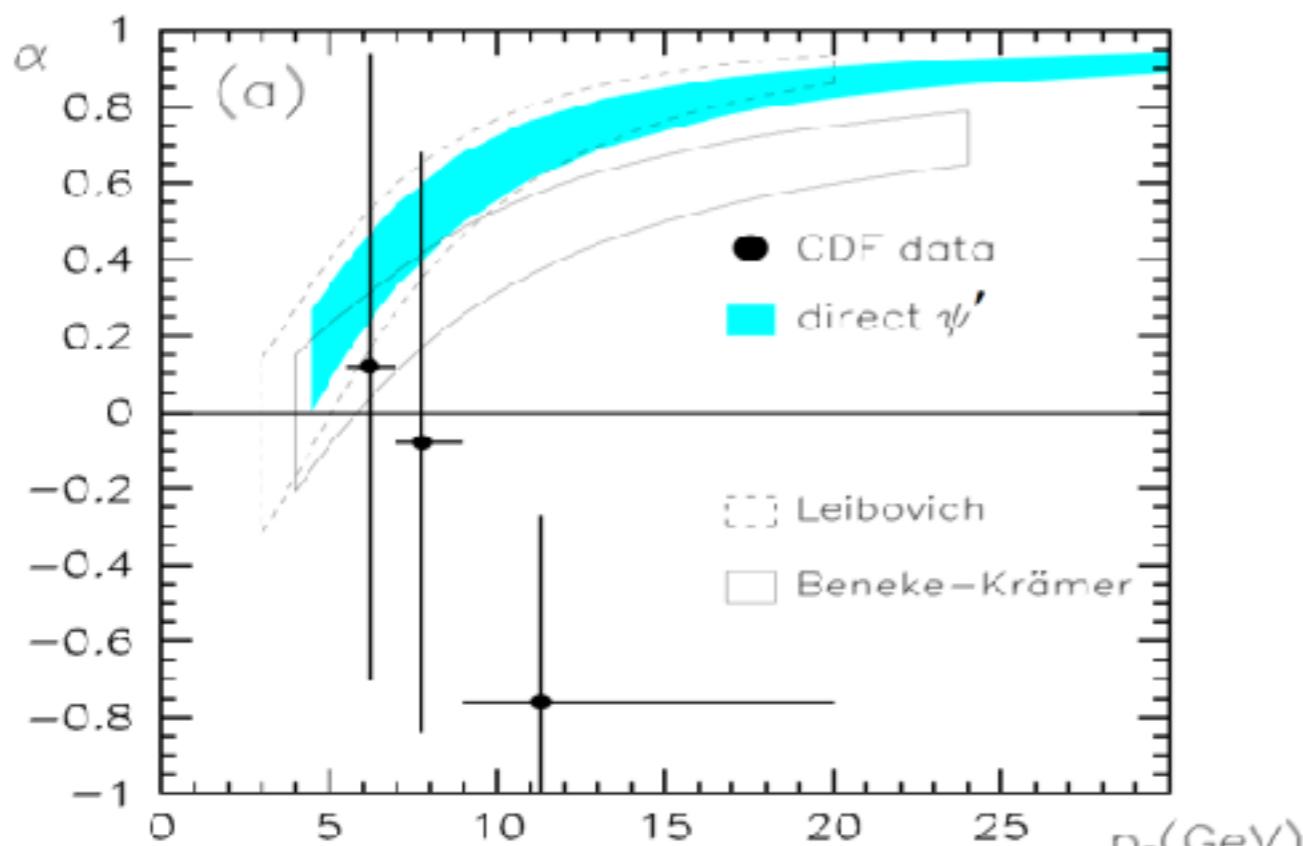
# Color-Octet $^3S_1$ fragmentation function, FJF

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003

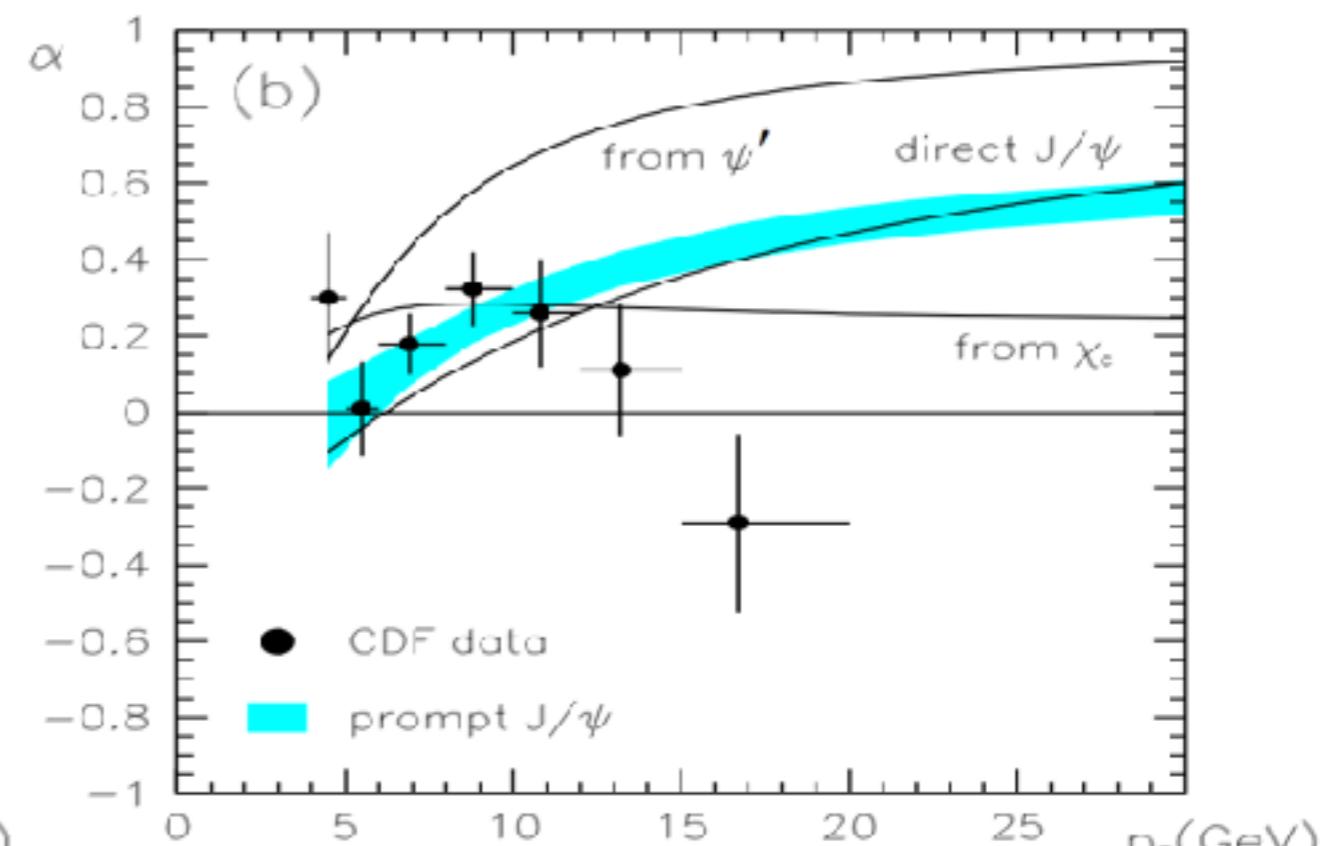


# Polarization Puzzle

$^3S_1^{[8]}$  fragmentation at large  $p_T$  predicts transversely polarized  $J/\psi, \psi'$



$\psi'$



$J/\psi$

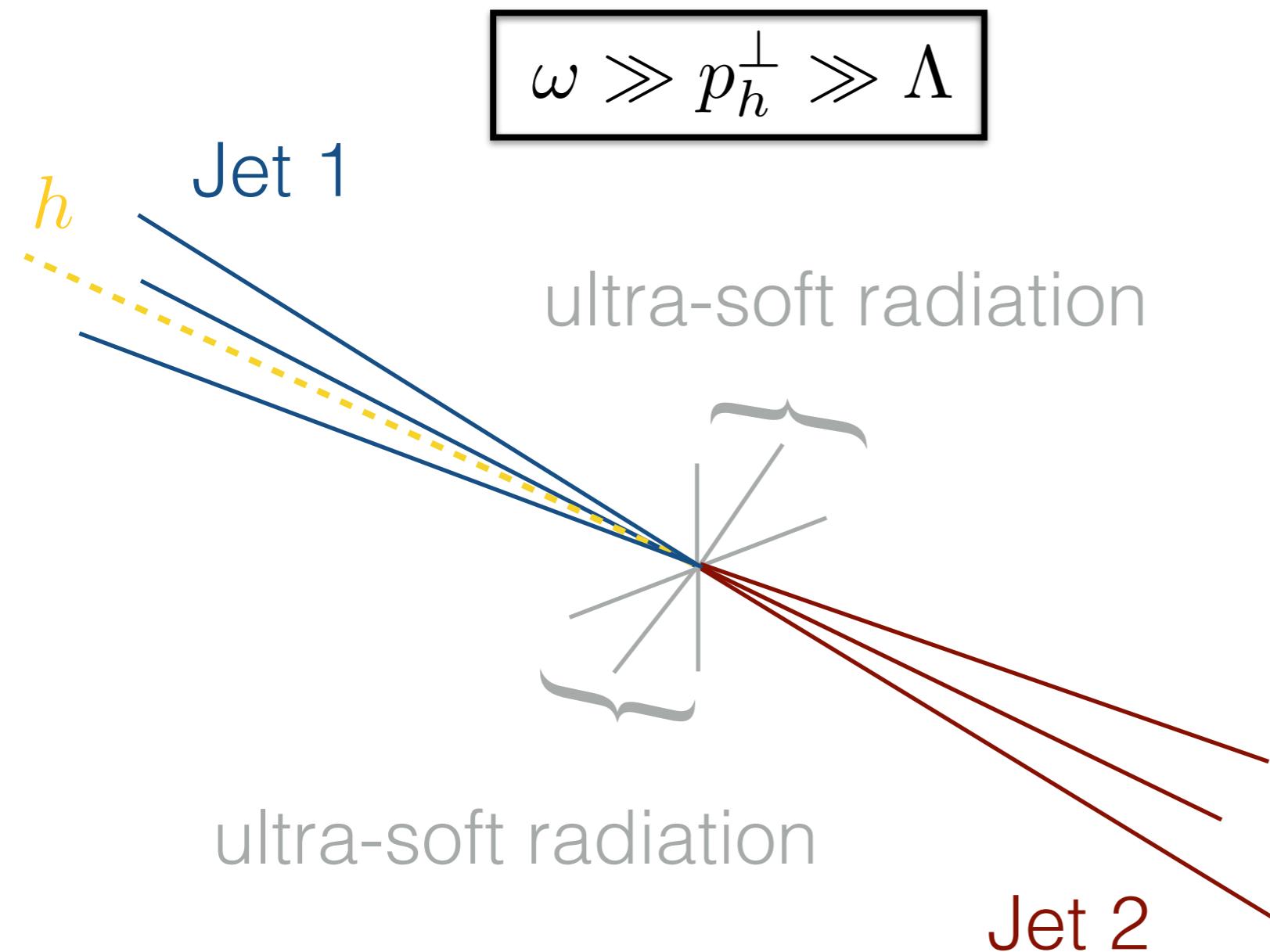
Braaten, Kniehl, Lee, 1999

$$D_{q/h}(\mathbf{p}_\perp,z,\mu)=\frac{1}{z}\sum_X \frac{1}{2N_c}\delta(p^-_{Xh;r})\delta^{(2)}(\mathbf{p}_{\textcolor{brown}{L}}+\mathbf{p}^X_\perp)\,\text{Tr}\left[\frac{\not{\epsilon}}{2}\langle 0|\delta_{\omega,\overline{\mathcal{P}}}\chi_n^{(0)}(0)|Xh\rangle\right.\nonumber\\ \left.\langle Xh|\bar{\chi}_n^{(0)}(0)|0\rangle\right]$$

$$\int d^2\mathbf{p}^h_\perp\; D_{q/h}(\mathbf{p}^h_\perp,z,\mu) = D_{q/h}(z,\mu)$$

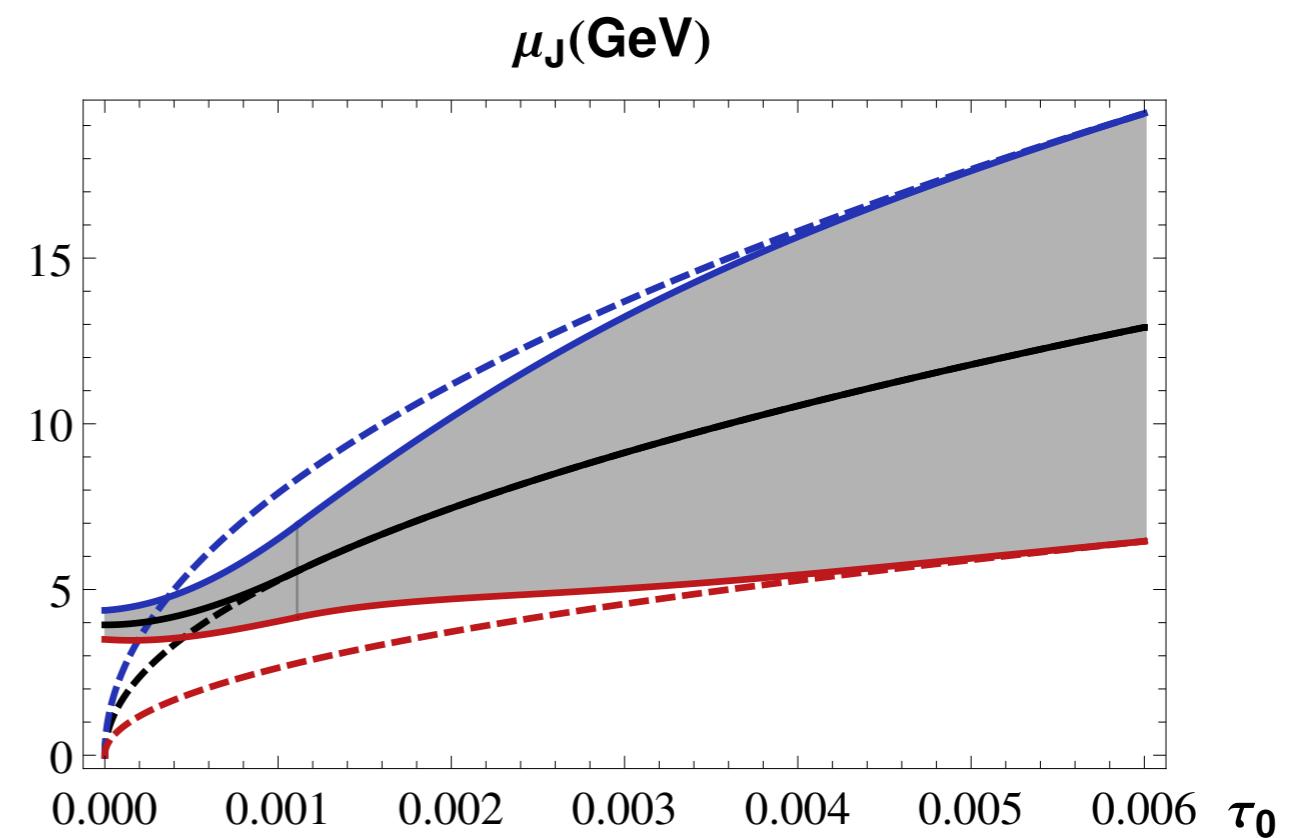
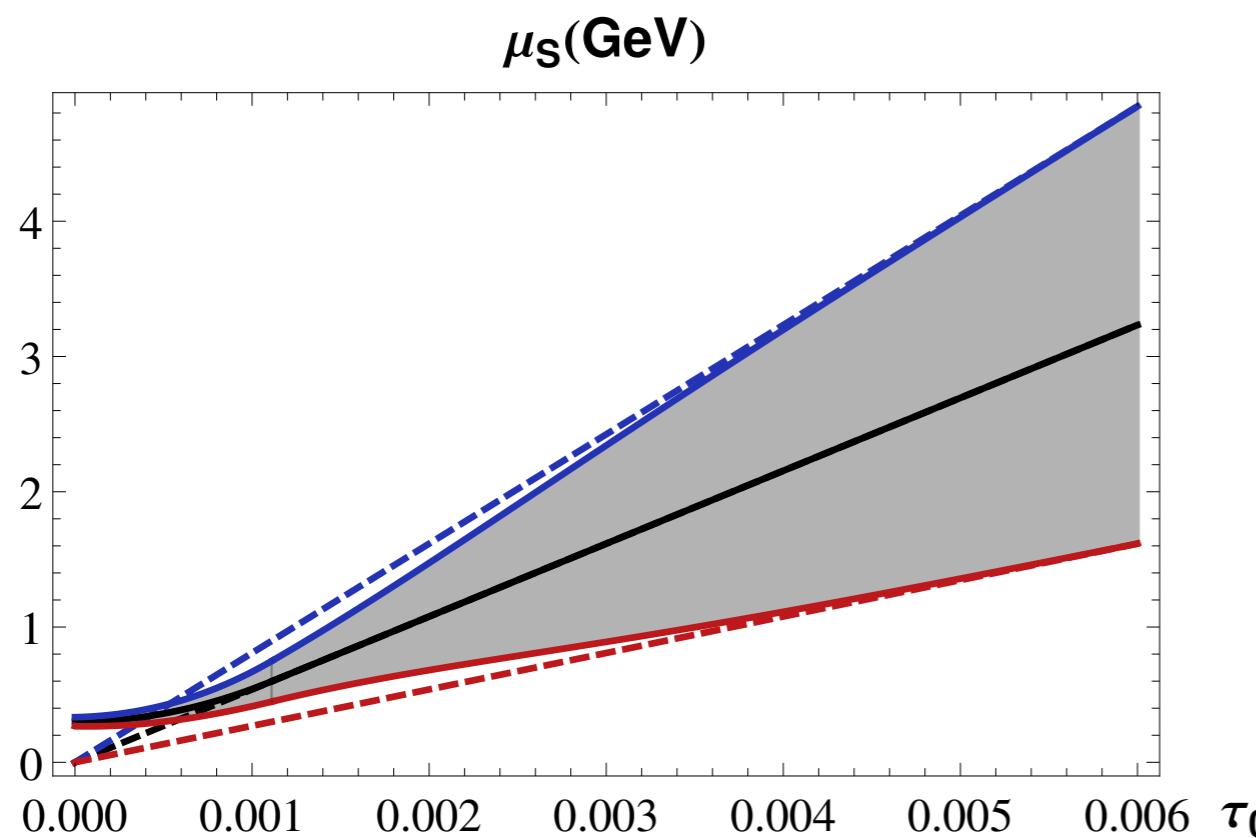
# Transverse Momentum Dependent FJFs

R. Bain, Y. Makris, TM, JHEP 1611 (2016) 144

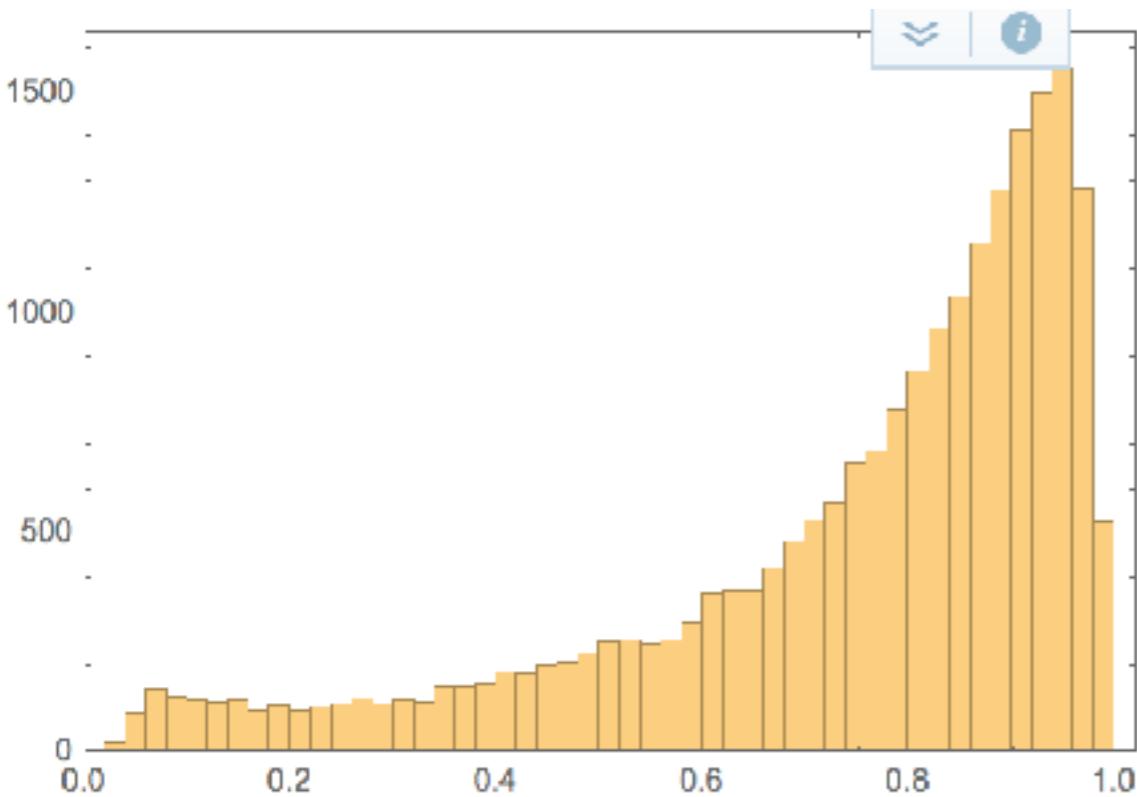


$D_{i/h} (z, p_h^\perp, \mu)$
$p_c \sim \omega(\lambda^2, 1, \lambda)$
$p_{cs} \sim p_h^\perp(r, 1/r, 1)$
$p_{us} \sim \Lambda(1, 1, 1)$
$\lambda = p_h^\perp/\omega$

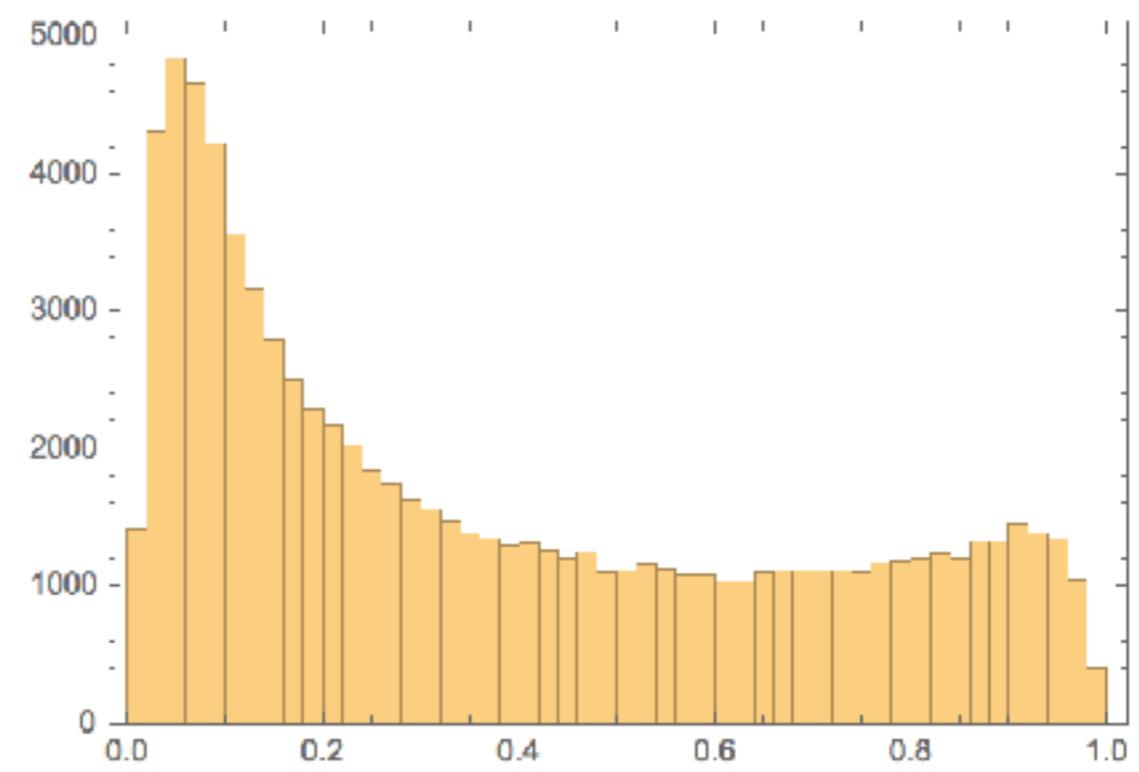
# Profile Functions



	Traditional	Profile
<b>Canonical</b>	-----	—
$\epsilon_{S/J}=+1/2 (+50\%)$	- - -	—
$\epsilon_{S/J}=-1/2 (-50\%)$	- - -	—



**c distribution**



**g distribution**